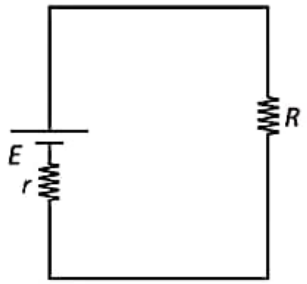
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. In a circuit there is a battery with internal resistance r and Emf E, which is connected to external load resistance R as shown. Find value of R so that maximum power dissipates across R.



(1) $R = r$

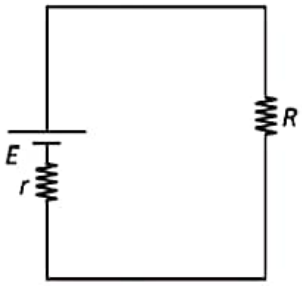
(2) $R = r/2$

(3) $R = \sqrt{2}r$

(4) $R = 2r$

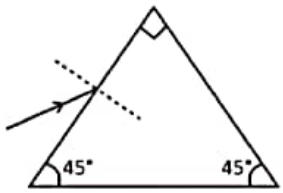
Answer (1)

Sol.



Maximum power transfer occurs for $R = r$.

2. Refractive index of prism is $\sqrt{2}$. What should be angle of incidence for a light ray such that the emerging ray grazes out of the surface.



(1) 90°

(2) 60°

(3) 30°

(4) 45°

Answer (1)

Sol. $r_1 + r_2 = 90°$

$$\sqrt{2} = \frac{\sin i}{\sin r_1}$$

and $$\sqrt{2} = \frac{1}{\sin r_2}$$

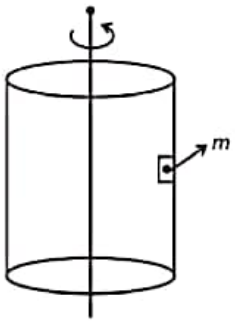
$$\sin r_2 = \frac{1}{\sqrt{2}}$$

$r_2 = 45°$

$r_1 = 45°$

$\therefore \quad i = 90°$

3. A block of mass m is at rest w.r.t. hollow cylinder which is rotating with angular speed ω, radius of cylinder is R. Find minimum coefficient of friction between block and cylinder.



(1) $\frac{3g}{2\omega^2 R}$

(2) $\frac{g}{\omega^2 R}$

(3) $\frac{g}{4\omega^2 R}$

(4) $\frac{2g}{\omega^2 R}$

Answer (2)

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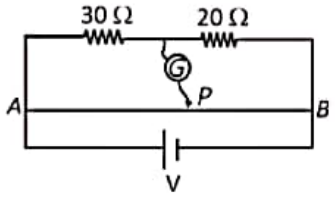
Sol. $\mu m\omega^2 R = mg$

$$\mu = \frac{g}{\omega^2 R}$$

4. In a meter bridge two balancing resistances are 30 Ω and 20 Ω. If galvanometer shows zero deflection for the jockey's contact point *P*. Then find the length A.P.



(1) 40 cm (2) 30 cm

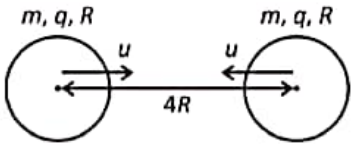
(3) 60 c, (4) 70 cm

Answer (3)

Sol. $\frac{30}{20} = \frac{\ell}{100-\ell}$

$\ell = 60$ cm

5. Two spheres having equal mass *m*, charge *q* and radius *R*, are moving towards each other. Both have speed *u* at an instant when distance between their centers is 4*R*. Minimum value of *u* so that they touch each other is



(1) $\sqrt{\frac{q^2}{4\pi\varepsilon_0 mR}}$ (2) $\sqrt{\frac{q^2}{16\pi\varepsilon_0 mR}}$

(3) $\sqrt{\frac{q^2}{\pi\varepsilon_0 mR}}$ (4) $\sqrt{\frac{q^2}{8\pi\varepsilon_0 mR}}$

Answer (2)

Sol. Energy conservation,

$$2\times\frac{1}{2}mu^2 + \frac{Kq^2}{4R} = \frac{Kq^2}{2R}$$

$$\Rightarrow mu^2 = \frac{Kq^2}{4R}$$

$$\Rightarrow u = \sqrt{\frac{Kq^2}{4mR}} = \sqrt{\frac{q^2}{16\pi\varepsilon_0 mR}}$$

6. RMS speed for H_2 and O_2 are same. If temperature of O_2 gas is 23°C. Find temperature of H_2 gas.

(1) 18.5 K (2) 2.5°C

(3) 18°C (4) 164 K

Answer (1)

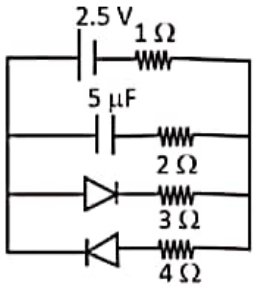
Sol. $V = \sqrt{\frac{3RT}{M}}$

$$T_{H_2} = T_{O_2}\frac{M_{H_2}}{M_{O_2}}$$

$$= 296\times\frac{1}{16}$$

$$T_{H_2} = 18.5\,K$$

7. For the given circuit arrangement, find the charge on the capacitor in steady state.



(1) 5 μF (2) $\frac{75}{8}\mu C$

(3) $\frac{15}{2}\mu F$ (4) $\frac{55}{4}\mu C$

Answer (2)

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Sol. Forward biased diode will be allowing current. So current through battery $I=\frac{2.5}{4}=\frac{5}{8}A$

So ΔV across capacitor is

$$\Delta V=\left(\frac{5}{8}\right)\times 3=\frac{15}{8}\text{ volt}$$

So, $q=\frac{5\times 15}{8}=\frac{75}{8}\mu C$

8. An electron in a hydrogen like atom has energy equal to $-0.04\,E_0$, where E_0 is magnitude of energy of this electron in ground state in eV. If angular momentum of this electron is L, then value of $\frac{2\pi L}{h}$ is ($h \rightarrow$ Planck's constant)

(1) 1 (2) 4

(3) 5 (4) 6

Answer (3)

Sol. $13.6\,Z^2 = E_0$

and $\frac{-13.6\,Z^2}{n^2}=-0.04\,E_0$

$$n^2=\frac{1}{0.04}=25$$

$$n = 5$$

$$\frac{2\pi L}{n}=\frac{2\pi}{n}\frac{nh}{2\pi}=5$$

9. During SHM, K.E. of particle in SHM varies with frequency of 176 Hz. Find out frequency of SHM of the particle.

(1) 352 (2) 176

(3) 88 (4) 44

Answer (3)

Sol. Conceptual.

KE varies with twice the frequency of SHM.

10. Position x of the particle of mass 2kg varies as function of time as $x = t^2 + t + 1$. Find out work done on the particle from $t_1 = 2$ sec to $t_2 = 3$ sec.

(1) 18 joule (2) 30 joule

(3) 34 joule (4) 24 joule

Answer (4)

Sol. $\Delta W = K.E_f - K.E_i$

$$v=\frac{dx}{dt}=2t+1$$

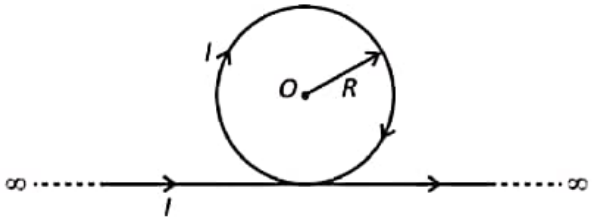
So, $V_{(1)} = 5$ m/s

$V_{(2)} = 7$ sec.

$$\Delta\omega=\frac{1}{2}\times 2\left|V_2^2-V_1^2\right|$$

$\Delta w = 49 - 25 = 24$ Joule

11. Find magnetic field at point 'O' in the given figure shown.



(1) $\frac{\mu_0 I}{R}\left(2+\frac{1}{\pi}\right)$ (2) $\frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)$

(3) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)$ (4) $\frac{\mu_0 I}{4R}\left(2+\frac{1}{\pi}\right)$

Answer (3)

Sol. $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)$

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12. Two physical quantities x and y has measured value 23.01 and 37.3 respectively. Then value of $x + y$ is

(1) 60.3 (2) 60.31

(3) 60.03 (4) 60.310

Answer (1)

Sol. 23.01 + 37.3 = 60.31

= 60.3

13. Consider two statements given below :

Statement-I: In YDSE, if distance between slits & screen increases, fringe width also increases.

Statement-II: If wavelength of light used in YDSE increases, fringe width also increases.

Which of the following options is correct.

(1) Both statements I & II are correct

(2) Statement I is correct but statement II is incorrect

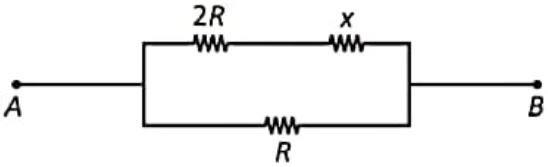
(3) Statement I is incorrect but statement II is correct

(4) Both statements I & II are incorrect

Answer (1)

Sol. $\beta = \frac{\lambda D}{d}$

14. For the given resistive network, the net resistance across $AB = x$. Then find the value of x.



(1) $x = R(\sqrt{2}-1)$ (2) $x = R(\sqrt{3}+1)$

(3) $x = R(\sqrt{2}+1)$ (4) $x = R(\sqrt{3}-1)$

Answer (4)

Sol. $R_{(AB)} = \frac{R(2R+x)}{R+2R+x} = x$

$2R^2 + Rx = 3Rx + x^2$

$x^2 + 2Rx - 2R^2 = 0$

$x = R(\sqrt{3}-1)$

15. Work done in a isobaric process is 100 Joule. If adiabatic constant for the gas is 1.4. Find heat given to the gas.

(1) 250 J (2) 350 J

(3) 150 J (4) 160 J

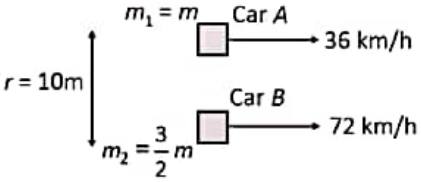
Answer (2)

Sol. $\frac{\Delta U}{W} = \frac{C_V}{R} = \frac{5}{2}$

$\Delta U = 250$

$Q = 350$ J

16. Two cars moving parallelly to each other with the velocities shown. Initial separation between cars is r = 10m. Find angular momentum of car A w.r.t. car B



(1) 100 m (2) 200 m

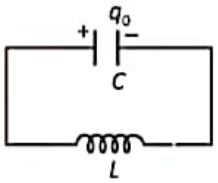
(3) 150 m (4) 75 m

Answer (1)

Sol. $L_{A/B} = m(36-72)(10) \times \frac{5}{18}$

= 100 m

17. In the given L-C circuit, charge on the capacitor is maximum at $t = 0$, find time at which charge becomes 25% of it's initial value first time.



(1) $\sqrt{LC}\cos^{-1}\left(\frac{1}{4}\right)$ (2) $\frac{L}{R}\ln 2$

(3) $\sqrt{LC}\sin^{-1}\left(\frac{1}{4}\right)$ (4) $\sqrt{LC}\cos^{-1}\left(\frac{1}{2}\right)$

Answer (1)

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Sol. $q = q_0\cos(\omega t)$

$$\frac{q_0}{4} = q_0\cos(\omega t)$$

$$\frac{1}{\sqrt{LC}}t = \cos^{-1}\left(\frac{1}{4}\right)$$

18. A boat crosses a river, 200 m wide, in minimum possible time. If velocity of river is 5 m/s and velocity of boat is still water is 10 m/s. Then, find time taken to cross the river and displacement of the boat.

(1) 20 sec. and $100\sqrt{5}$ m

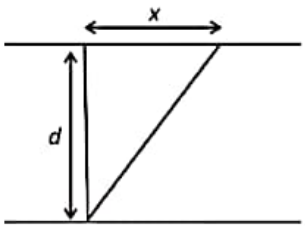
(2) 10 sec. and $100\sqrt{5}$ m

(3) 20 sec. and $200\sqrt{5}$ m

(4) 20 sec. and 200 m

Answer (1)

Sol.



$$10 = \frac{200}{t_{min}}$$

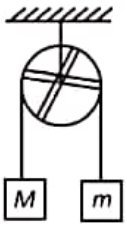
t_{min} = 20 sec.

$x = 5 \times 20 = 100$ m

displacement = $\sqrt{100^2 + 200^2}$

$= 100\sqrt{5}$

19. In diagram given below, pulley is a ring of mass M radius R fitted with two rods each of mass m & length $2R$ along diameter such that if pulley rotates, Rods also rotate with same angular velocity.



Find magnitude of acceleration of m when system is released.

(1) $\frac{3(M-m)g}{(6M+5m)}$ (2) $\frac{6(M-m)g}{(6M+5m)}$

(3) $\frac{3(M-m)g}{(M+m)}$ (4) $\frac{6(M-m)g}{(M+m)}$

Answer (1)

Sol. $(M-m)gR = \left(MR^2 + mR^2 + MR^2 + \frac{2}{3}mR^2\right)\alpha$

$$a = \frac{3(M-m)g}{6M+5m}$$

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21.

22.

23.

24.

25.

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CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Given below are two statements :

   Statement I : The correct order for radius is Al > Mg > Mg^{2+} > Al^{3+}.

   Statement II : Atomic size always depends on electronegativity.

   In the light of the above statements, choose the correct option.

   (1) Both Statement I and Statement II are correct

   (2) Both Statement I and Statement II are incorrect

   (3) Statement I is correct but Statement II is incorrect

   (4) Statement I is incorrect but Statement II is correct

Answer (3)

Sol. Atomic radius : Mg > Al > Mg^{2+} > Al^{3+}

Atomic radius depends on Z_{eff}, number of shells etc.

2. What will be significant figure of summation of 0.153, 153.2 and 1532?

   (1) 3

   (2) 4

   (3) 5

   (4) 6

Answer (2)

Sol. 1532 + 153.2 + 0.153 = 1685.353 = 1685 (least decimal = 0)

3. Given below are two statements :

   Statement-I : Crystal field stabilisation energy (magnitude) of $[Co(H_2O)_6]^{2+}$ is greater than $[Ni(H_2O)_6]^{2+}$

   Statement-II : Order of bond energy is Cl_2 > Br_2 > F_2 > I_2.

   In the light of above statements choose the correct option.

   (1) Statement-I and Statement-II both are correct

   (2) Statement-I and Statement-II both are incorrect

   (3) Statement-I is correct, Statement-II is incorrect

   (4) Statement-I is incorrect, Statement-II is correct

Answer (4)

Sol. $[Co(H_2O)_6]^{2+}$

H_2O is WFL with Co^{2+}

$3d^7 \Rightarrow t_{2g}^5 e_g^2$

CFSE = $-0.4 \times 5\Delta_o + 2 \times 0.6\ \Delta_o$

= $-2.0\Delta_o + 1.2\Delta_o$

= $-0.8\Delta_o$

$[Ni(H_2O)_6]^{2+}$

$H_2O \Rightarrow$ WFL

$3d^8 \Rightarrow t_{2g}^6 e_g^2$

CFSE = $-0.4 \times 6\Delta_o + 0.6 \times 2\ \Delta_o$

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$= -2.4\Delta_o + 1.2\Delta_o$

$= -1.2\ \Delta_o$

(BDE in kJ/mol)

$F_2 \Rightarrow 158.8$

$Cl_2 = 242.6$

$Br_2 = 192.8$

$I_2 = 151.1$

Order of BDE $\Rightarrow Cl_2 > Br_2 > F_2 > I_2$

4. When 8.74 g MnO_2 is treated with HCl, then what will be the weight of $Cl_2(g)$ obtained?

Molar mass of MnO_2 = 87.4 g/mol

(1) 7.1 g

(2) 17.1 g

(3) 14.2 g

(4) 3.55 g

Answer (1)

Sol. $MnO_2 + 4HCl \rightarrow MnCl_2 + Cl_2 + 2H_2O$

$\frac{8.74}{87.4} = 0.1$ 0.1 mol

$M_{Cl_2} \cong 7.1\,g$

5. Find concentration of X^{2-} at equilibrium in 0.1 M H_2X.

Given $K_{a_1} = 2.5 \times 10^{-7}$

$K_{a_2} = 1 \times 10^{-13}$

(1) 2.5×10^{-7}

(2) 1×10^{-13}

(3) 6×10^{-12}

(4) 5×10^{-10}

Answer (2)

Sol.

	H_2X	$\rightleftharpoons$	H^+	+	HX^-
$t=0$	C		0		0
$t=t_{eq}$	$C(1-\alpha)$		$C\alpha + Y$		$C\alpha$

	HX^-	$\rightleftharpoons$	H^+	+	X^{2-}
	$C\alpha$				
	$C\alpha - Y$		$C\alpha + Y$		Y

$$K_{a_2} = \frac{[H^+][X^{2-}]}{[HX^-]} = \frac{(C\alpha + Y)(Y)}{(C\alpha - Y)}$$

Since K_{a_2} is very small

Hence $Y \approx 0$ $C\alpha >> Y$

$K_{a_2} = [X^{2-}]$

$[X^{2-}] = 1 \times 10^{-13}$

6. What will be the ratio of wavelength of 3rd line of Paschen Series to 2nd line of Balmer series of H-atom?

(1) $\frac{9}{4}$

(2) $\frac{3}{2}$

(3) $\frac{2}{3}$

(4) $\frac{16}{4}$

Answer (1)

Sol. $\frac{\frac{1}{\lambda_P}}{\frac{1}{\lambda_B}} = \frac{\left(\frac{1}{3^2}\right) - \left(\frac{1}{6^2}\right)}{\left(\frac{1}{2^2}\right) - \left(\frac{1}{4^2}\right)} \Rightarrow \frac{4}{9}$

$\frac{\lambda_P}{\lambda_B} = \frac{9}{4}$

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7. $K_2Cr_2O_7$ is heated with KCl in pressure of H_2SO_4. Find the correct match of product with their oxidation state.

(1) CrO_2Cl_2, +6

(2) $Cr_2O_2Cl_2$, +6

(3) $Cr_2O_2Cl_2$, +5

(4) CrO_2Cl_2, +5

Answer (1)

Sol. When $K_2Cr_2O_7$ is heated with KCl in pressure of H_2SO_4, deep red vapours of CrO_2Cl_2 is observed. In CrO_2Cl_2, Cr is in +6 state.

8. Osmotic pressure of a solution is 12 atm. What is the concentration of NaCl solution which is isotonic to the given solution at 900 K.

$R = 0.082$ L-atm K^{-1} mol^{-1}

(Assume 100% dissociation)

(1) 0.4878 M

(2) 0.0243 M

(3) 0.243 M

(4) 0.04878 M

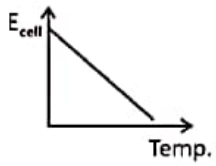
Answer (3)

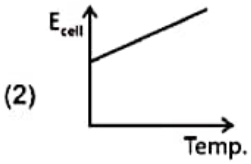
Sol. $12 = i \times C \times 0.082 \times 300$

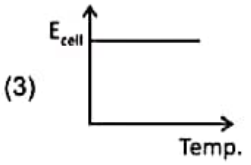
$$\frac{12}{2\times0.082\times300} = C$$

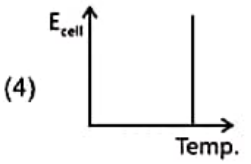
$C = 0.243$ M

9. Find out correct graph

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. $E_{cell} = E^{\circ}_{cell} - \frac{RT}{nF}\log q$

Temp ↑ E_{cell} ↓

10. $CH_3Br \xrightarrow{Nu^{\ominus}}$

Order of reactivity of nucleophiles given below,

$^{\ominus}OH$ $CH_3COO^{\ominus}$ $PhO^{\ominus}$ $ClO_4^{\ominus}$

(1) $Ph{-}O^{\ominus} > {}^{\ominus}OH > ClO_4^{\ominus} > CH_3COO^{\ominus}$

(2) ${}^{\ominus}OH > Ph{-}O^{\ominus} > CH_3{-}COO^{\ominus} > ClO_4^{\ominus}$

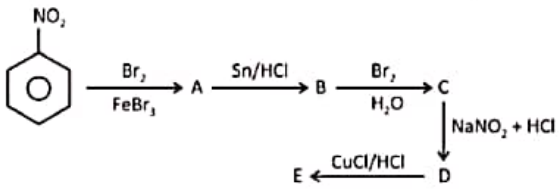
(3) $ClO_4^{\ominus} > {}^{\ominus}OH > Ph{-}O^{\ominus} > CH_3COO^{\ominus}$

(4) $CH_3COO^{\ominus} > {}^{\ominus}OH > PhO^{\ominus} > ClO_4^{\ominus}$

Answer (2)

Sol. ${}^{\ominus}OH > Ph{-}O^{\ominus} > CH_3{-}COO^{\ominus} > ClO_4^{\ominus}$

11. Observe the following reaction sequence,



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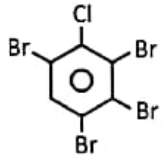
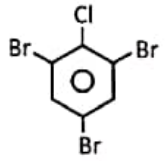
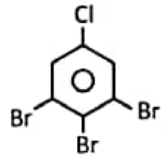
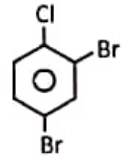
The final product 'E' is

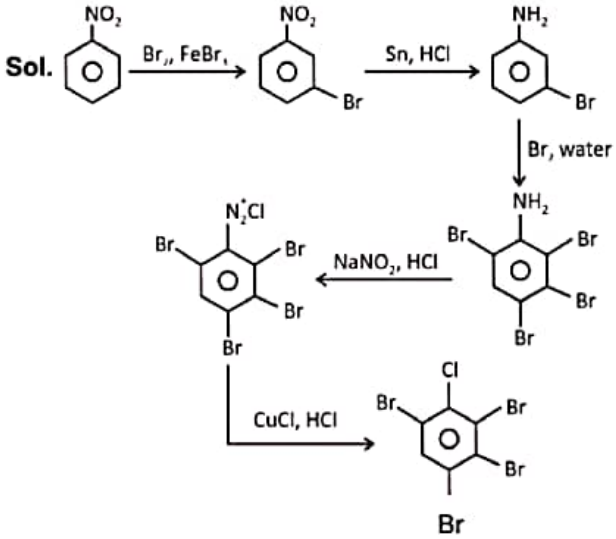
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

12. For first order kinetics reaction,

$A \rightarrow B + C$

If initial pressure of A is 1 bar and at time 100 s, the total pressure is 1.5 bar, then find the rate constant of the reaction.

(1) $6.93 \times 10^{-3}\ s^{-1}$

(2) $6.93 \times 10^{-2}\ s^{-1}$

(3) $0.693\ s^{-1}$

(4) $6.93\ s^{-1}$

Answer (1)

Sol.

	A	$\rightarrow$	B	+	C
$t=0$	P_0		–		–
$t=100\ s$	$P_0 - x$		x		x

$P_t = P_0 + x$

$x = P_t - P_0$

$$k = \frac{2.303}{100}\log\frac{P_0}{P_0 - P_t + P_0}$$

$$k = \frac{2.303}{100}\log\frac{P_0}{2P_0 - P_t}$$

$$k = \frac{2.303}{100}\log\frac{1}{2-1.5}$$

$$k = \frac{2.303}{100}\log 2 = \frac{2.303\log 2}{100}$$

$= 0.693 \times 10^{-2}$

$= 6.93 \times 10^{-3}\ s^{-1}$

13. Energy of first line of Lyman series – A

Energy of second line of Balmer series – B

Energy of first line of Balmer series – C

Energy of second line of Lyman series – D

What will be the correct decreasing order of energies of photons?

(1) C > A > B > D

(2) D < A > B > C

(3) D > A > C > B

(4) D > A > B > C

Answer (4)

Sol. A . Lyman/1st line $\Delta E = 13.6 - 3.4 = 10.2$ eV

B. Balmer/2nd line $\Delta E = 3.4 - 0.85 = 2.55$ eV

C. Balmer/1st line $\Delta E = 3.4 - 1.51 = 1.89$ eV

D. Lyman/2nd line $\Delta E = 13.6 - 1.51 = 12.09$ eV

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14. Which compound is optically inactive out of following

n-propyl chloride (I), secondary butyl chloride (II),

tert butyl chloride (III), isopropyl chloride (IV).

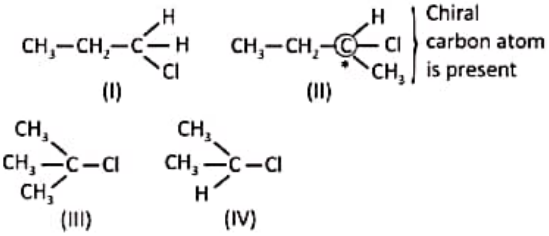
(1) Only I, III, IV

(2) Only IV

(3) Only I, II, III

(4) Only II, III, IV

Answer (1)

Sol. 

(I) (II)

(III) (IV)

15. Which of the following statements are true?

(i) Mn has highest oxidation state in Mn_2O_7

(ii) MnO is more Ionic than Mn_2O_7

(iii) Mn_2O_7 has one bridging O atom

(iv) Oxidation state of Mn is generally maximum in oxo compounds

(1) Only (i), (ii), (iii) are correct

(2) All (i), (ii), (iii) and (iv) are correct

(3) Only (i), (iii) and (iv) are correct

(4) Only (i) and (iv) are correct

Answer (2)

Sol. (i) $Mn_2O_7 \rightarrow$ Mn is +7 (oxidation state)

(ii) MnO is more Ionic than Mn_2O_7.

(iii) Each Mn is tetrahedrally surrounded by 4 oxygen atom and one oxygen is bridging (Mn-O-Mn)

(iv) In Mn_2O_7 the Mn is in +7 oxidation state (Maximum)

16. Match the two columns

	List-I (Name reaction)		**List-II Reagent(s)**
(A)	Etard reaction	(i)	$H_2/Pd\text{-}BaSO_4$
(B)	Gattermann Koch reaction	(ii)	(a) $SnCl_2$ + HCl (b) H_3O^+
(C)	Stephen reaction	(iii)	CO + $HCl/AlCl_3$(anhy)
(D)	Rosenmund reduction	(iv)	(a) CrO_2Cl_2/CS_2 (b) H_3O^+

Choose the correct answer:

(1) A – iv, B – ii, C – iii, D – i

(2) A – iv, B – iii, C – ii, D – i

(3) A – iv, B – iii, C – i, D – ii

(4) A – iv, B – i, C – iii, D – ii

Answer (2)

Sol.

Etard reaction	CrO_2Cl_2/H^+
Gattermann Koch	CO + $HCl/AlCl_3$
Stephen reaction	$SnCl_2$ + HCl
Rosenmund reduction	H_2, Pd–$BaSO_4$

17. In which of the following pairs first compound have more covalent nature than second compound?

(a) $SnCl_2$, $SnCl_4$

(b) $PbCl_4$, $PbCl_2$

(c) UF_6, UF_4

(1) Only (a) and (b)

(2) Only (b) and (c)

(3) Only (a) and (c)

(4) Only (c)

Answer (2)

Sol. More the charge on cation more will be polarising power and more will be covalent character.

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18. Solubility product of MX(s) is 10^{-10} and $E^{\circ}_{M^+/M} = 0.71\,V$. Find out $E^{\circ}_{M/MX/X^-}$.

(1) 0.119 V (2) –0.119 V

(3) 1.301 V (4) –1.301 V

Answer (2)

Sol. $E^{\circ}_{M/MX/X^-} = -E^{\circ}_{M^+/M} - \frac{0.0591}{1}\log K_{sp}$

$= -0.71 - 0.0591(-10)$

$= -0.71 + 0.591$

$= -0.119\,V$

19. 5 g of a solute X (M. wt = 200 g/mol) is dissolved in 250 g benzene. If $\Delta T_f = 0.5$ K and relative lowering of vapour pressure is 'P' find $P \times 10^4$

$K_f = 5.5\ K\ kg\ mol^{-1}$, solute dimerises in benzene.

(1) 253.6 (2) 0.1636

(3) 70 (4) 23.36

Answer (3)

Sol. $0.5 = i \times 5.5 \times \frac{5}{200} \times \frac{1000}{250}$

$i = \frac{0.5 \times 10}{5.5}$

$= 0.91$

$1 - \frac{\alpha}{2} = 0.91 \qquad 1 - 0.91 = \frac{\alpha}{2}$

$0.09 \times 2 = \alpha$

$0.18 = \alpha$

$\frac{P^o - P_s}{P^o} = \frac{i\ n_{solute}}{i \times n_{solute} + n_{solvents}}$

$= \frac{i \times 5/200}{i \times \frac{5}{200} + \frac{250}{78}} = \frac{0.91\quad 0.025}{0.91 \times 0.025 + 3.20}$

$= 70 \times 10^{-4}$

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. 1 g of an organic compound produce 1.49g of $Mg_2P_2O_7$. Determine % of P

Answer (42)

Sol. Mass of P in organic compound

$= \frac{1.49}{222} \times 31 \times 2$

$= 0.4161$ g

% of P in organic compound

$= = \frac{0.4161}{1} \times 100$

$= 41.61\%$

$\approx 42\%$

22. Some species are given

Ni^{2+}, Fe^{2+}, Co^{2+}, V^{3+} and Ti^{2+}

How many species has magnetic moment (spin only) less than 3 BM.

Answer (3)

Sol. $\mu_{spinonly} = \sqrt{n(n+2)}$ BM

$Ni^{2+} \Rightarrow 3d^8 \Rightarrow$ $\underset{n=2}{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{1}\boxed{1}}$

$\mu = \sqrt{2(2+2)}\ BM = \sqrt{8} = 2.83\ BM$

$Fe^{2+} \Rightarrow 3d^6 \Rightarrow$ $\underset{n=4}{\boxed{\uparrow\downarrow}\boxed{1}\boxed{1}\boxed{1}\boxed{1}}$

$\mu = 4.90$

$Co^{2+} \Rightarrow 3d^7 \Rightarrow$ $\underset{n=3}{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{1}\boxed{1}\boxed{1}}$

$\mu = 3.87$

$V^{3+} \Rightarrow \mu = 2.83$ BM

$Ti^{2+} \Rightarrow \mu = 2.83$ BM

23.

24.

25.

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MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

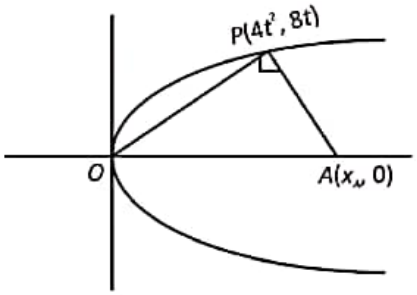
1. Let O be the vertex of the parabola $y^2 = 16x$. The locus of centroid of ΔOPA when P lies on parabola and A lies on x-axis and $\angle OPA = 90°$

(1) $y^2 = 8(3x - 16)$ (2) $9y^2 = 8(3x - 16)$

(3) $y^2 = 8(3x + 16)$ (4) $9y^2 = 8(3x + 16)$

Answer (2)

Sol.



$m_{OP} \cdot m_{PA} = -1$

$$\frac{2}{t} \cdot \frac{8t}{4t^2 - x_A} = -1$$

$-16 = 4t^2 - x_A$

$x_A = 4t^2 + 16$

$$h = \frac{4t^2 + 4t^2 + 16}{3} \qquad k = \frac{8t}{3}$$

$$h = \frac{8t^2 + 16}{3} \qquad t = \frac{3k}{8}$$

$$3h - 16 = 8\left[\frac{3k}{8}\right]^2$$

Replace (h, k) with (x, y)

$$3x - 16 = \frac{9y^2}{8}$$

$$\boxed{9y^2 = 8(3x - 16)}$$

2. If the product

$$\left(\frac{1}{15C_0} + \frac{1}{15C_1}\right)\left(\frac{1}{15C_1} + \frac{1}{15C_2}\right)\ldots\left(\frac{1}{15C_{12}} + \frac{1}{15C_{13}}\right)$$

$$= \frac{\alpha^{13}}{14C_0 \cdot {}^{14}C_1 \cdot {}^{14}C_2 \ldots {}^{14}C_{12}}$$, then 30α is equal to

(1) 16 (2) 32

(3) 15 (4) 28

Answer (2)

Sol. Notice that

$$\frac{1}{{}^nC_r} + \frac{1}{{}^nC_{r+1}} = \frac{{}^nC_{r+1} + {}^nC_r}{{}^nC_r \cdot {}^nC_{r+1}} = \frac{{}^{n+1}C_{r+1}}{{}^nC_r \cdot {}^nC_{r+1}}$$

$$= \frac{\frac{n+1}{r+1} + {}^nC_r}{{}^nC_r \cdot {}^nC_{r+1}} = \frac{(n+1)}{(r+1) \cdot \frac{n}{r+1} \cdot {}^{n-1}C_r} = \frac{n+1}{n \cdot {}^{n+1}C_r}$$

$$\therefore \quad \left(\frac{1}{{}^{15}C_0} + \frac{1}{{}^{15}C_1}\right)\left(\frac{1}{{}^{15}C_1} + \frac{1}{{}^{15}C_2}\right)\ldots\left(\frac{1}{{}^{15}C_{12}} + \frac{1}{{}^{15}C_{13}}\right)$$

$$= \frac{16}{15 \cdot {}^{14}C_0} \cdot \frac{16}{15 \cdot {}^{14}C_1} \ldots \frac{16}{15 \cdot {}^{14}C_{12}}$$

$$= \frac{\left(\frac{16}{15}\right)^{13}}{{}^{14}C_0 \cdot {}^{14}C_1 \cdot {}^{14}C_2 \ldots {}^{14}C_{12}}$$

$$\therefore \quad \alpha = \frac{16}{15}$$

$\therefore 30\alpha = 32.$

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3. If probability distribution is given by,

x	0	1	2	3
$p(n)$	$\frac{8a-1}{30}$	$\frac{4a-1}{30}$	$\frac{2a+1}{30}$	b

. If it is given that $\sigma^2+\mu^2=2$, where σ is standard derivation and μ is mean of distribution than $\frac{a}{b}$ is

(1) $\frac{22}{71}$ (2) $\frac{110}{71}$

(3) $\frac{220}{71}$ (4) $\frac{1110}{71}$

Answer (4)

Sol.

x	0	1	2	3
$p(n)$	$\frac{8a-1}{30}$	$\frac{4a-1}{30}$	$\frac{2a+1}{30}$	b

$\mu=\Sigma xP(n)$

$=\frac{(4a-1)}{30}+2\left(\frac{2a+1}{30}\right)+36=\frac{16a+90b}{30}$

$\sigma^2=\Sigma x^2P(n)-\mu^2$

$\sigma^2+\mu^2=\Sigma x^2P(n)=2$

$=\left(\frac{4a-1}{30}\right)(1)+\left(\frac{2a+1}{30}\right)4+9b=2$

$= 4a - 1 + 8a + 4 + 270b = 60$

$= 12a + 270b = 57$

$= 4a + 90b = 19$

$\sum P(n)=1\Rightarrow 14a-1+30b=30$

$14a + 90b = 31$

Solving we get $a=\frac{37}{19}, b=\frac{71}{570}$

$\frac{a}{b}=\frac{1110}{71}$

4. $\int_0^1 \cot^{-1}\left(x^2+x+1\right)dx$ is equal to

(1) $\int_0^1 \tan^{-1}(x+1)dx-\int_0^1 \tan^{-1}x dx$

(2) $\int_0^1 \left(\tan^{-1}(x+1)+\tan^{-1}x\right)dx$

(3) $\int_0^1 4\tan^{-1}x dx$

(4) $3\int_0^1 \tan^{-1}(x+1)dx$

Answer (1)

Sol. $I=\int_0^1 \cot^{-1}\left(x^2+x+1\right)dx$

$=\int_0^1 \tan^{-1}\left(\frac{1}{1+x(x+1)}\right)dx$

$=\int_0^1 \tan^{-1}\left(\frac{(x+1)-x}{1+(x+1)\cdot x}\right)dx$

$=\int_0^1 \left(\tan^{-1}(1+x)-\tan^{-1}x\right)dx$

$=\int_0^1 \tan^{-1}(1+x)dx-\int_0^1 \left(\tan^{-1}x\right)dx$

5. Let $f(x)=\lim_{n\to\infty}\left(\frac{1}{n^3}\sum_{k=1}^{n}\left[\frac{k^2}{3^n}\right]\right)$, where [.] denotes the greatest integer function, then $12\sum_{j=1}^{\infty}f(j)$ is equal to

(1) 2 (2) 3

(3) 4 (4) 1

Answer (1)

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Sol. $f(x)=\lim_{n\to\infty}\left(\frac{1}{n^3}\cdot\sum_{k=1}^{n}\left[\frac{k^2}{3^x}\right]\right)$

$$=\lim_{n\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}\left(\frac{k^2}{3^x}\right)-\lim_{n\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}\left\{\frac{k^2}{3^x}\right\}$$

$$f(x)=\lim_{n\to\infty}\frac{1}{n^3}\times\frac{n(n+1)(2n+1)}{6\times 3^x}-0$$

$$\Rightarrow\ f(x)=\frac{1}{3^{x+1}}$$

$$12\sum_{j=1}^{\infty}f(j)=12\left(\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\ldots\right)$$

$$=12\left(\frac{\frac{1}{9}}{1-\frac{1}{3}}\right)=\frac{12}{9-3}=2$$

6. The number of solutions of equation $\tan 3x = \cot x$ in $x\in[0,2\pi]$ is

(1) 4 (2) 6

(3) 2 (4) 8

Answer (4)

Sol. $\tan 3x=\cot=\tan\left(\frac{\pi}{2}-x\right)$

$$\Rightarrow 3x=\frac{\pi}{2}-x+n\pi, n\in I$$

$$\Rightarrow 4x=\frac{\pi}{2}+n\pi, n\in I$$

$$\Rightarrow x=\frac{\pi}{8}+\frac{n\pi}{4}, n\in I$$

$$\Rightarrow\frac{\pi}{8},\frac{3\pi}{8},\frac{5\pi}{8},\frac{7\pi}{8},\frac{9\pi}{8},\frac{11\pi}{8},\frac{13\pi}{8},\frac{15\pi}{8}$$

$= 8$

7. If $(\sec x)\frac{dy}{dx}-2y=2+3\sin x$ and $y(0)=-\frac{7}{4}$ then $y\left(\frac{\pi}{6}\right)$ is

(1) $\frac{3}{4}$ (2) $\frac{4}{3}$

(3) $\frac{5}{2}$ (4) $-\frac{5}{2}$

Answer (4)

Sol. $(\sec x)\frac{dy}{dx}-2y=2+3\sin x$

$$\Rightarrow\frac{dy}{dx}-2\cos xy=2\cos x+3\sin x\cdot\cos x$$

$IF = e^{-2\sin x}$

$$y\cdot e^{-2\sin x}=\int e^{-2\sin x}\cdot\cos x(2+3\sin x)dx$$

$\sin x = t \Rightarrow \cos x dx = dt$

$$y\cdot e^{-2\sin x}=\int e^{-2t}(2+3t)dt$$

$$=2\frac{e^{-2t}}{-2}+3\int t\cdot e^{-2t}dt$$

$$=-e^{-2t}+3\left(\frac{t\cdot e^{-2t}}{-2}+\frac{1}{2}\int e^{-2t}dt\right)$$

$$-e^{-2t}+3\left(-\frac{te^{-2t}}{2}-\frac{1}{4}e^{-2t}\right)+c$$

$$y=-1-\frac{3}{2}(\sin x)-\frac{3}{4}+ce^{2\sin x}$$

$$=-\frac{7}{4}-\frac{3}{2}\sin x+c\cdot e^{2\sin x}$$

$$y(0)=-\frac{7}{4}\Rightarrow c=0$$

$$y=-\frac{7}{4}-\frac{3}{2}\sin x$$

$$y(\pi/6)=-\frac{7}{4}-\frac{3}{4}=-\frac{10}{4}=-\frac{5}{2}$$

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8. The area bounded by

$2-4x \le y \le x^2+4$ and $x=\frac{1}{2}$

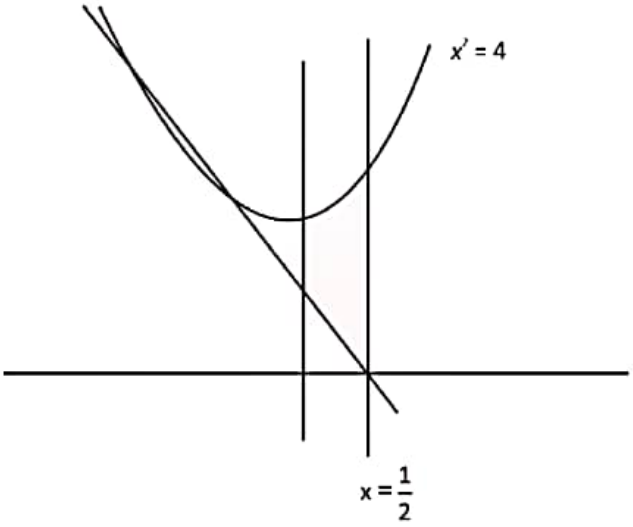
$x \ge 0$, $y \ge 0$ (in square unit) is

(1) $\frac{25}{37}$ sq. unit (2) $\frac{24}{37}$ sq. unit

(3) $\frac{37}{25}$ sq. unit (4) $\frac{37}{24}$ sq. unit

Answer (4)

Sol.



$$\text{Area} = \int_0^{\frac{1}{2}} (x^2+4)-(2-4x)dx$$

$$= \frac{x^3}{3}+2x+2x^2\Big]_0^{\frac{1}{2}} 1$$

$$\Rightarrow \frac{37}{24} \text{ sq unit}$$

9. Let A = {2, 3, 5, 7, 11} and a relation R is defined as $R=\{(x,y): x, y \in A, 2x \le 3y\}$. Then minimum number of elements are to be added to relation R such that R becomes symmetric relation is

(1) 4 (2) 8

(3) 7 (4)6

Answer (2)

Sol. $R=\{(x,y): 2x \le 3y\}$

{(2, 2), (3, 2)

(3, 3), (2, 3)

(5, 5), (2, 5), (3, 5), (7, 5)

(7, 7), (2, 7), (3, 7), (5, 7)

(11, 12), (2, 11), (3, 11), (5, 11), (7, 11)}

Since we want the relation to be symmetric relation,

We need to add

(5, 2), (5, 3), (7, 2), (7, 3), (11, 2), (11, 3), (11, 5), (11, 7)

$\Rightarrow$ 8 elements need to be added.

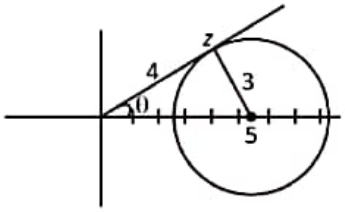
10. Let z be the complex number satisfying $|z-5| \le 3$ and having maximum possible positive argument, then the value of $34\left|\frac{5z-12}{5iz+16}\right|^2$ is equal to

(1) 20 (2) 17

(3) 7 (4) 21

Answer (1)

Sol.



$$\Rightarrow \arg(z) = \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right).$$

$$\Rightarrow z = |z|^{i\theta}$$

$$= 4 \times (\cos\theta + i\sin\theta)$$

$$= 4\times\left(\frac{4}{5}+i\frac{3}{5}\right)$$

$$= \frac{16}{5}+\frac{12i}{5} \qquad \Rightarrow 5z = 16+12i$$

$$5zi = 16i-12$$

$$\left(\frac{5z-12}{5zi+16}\right) = \frac{(4+12i)}{(4+16i)} = \frac{(1+3i)}{(1+4i)}.$$

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$$\Rightarrow \left|\frac{5z-12}{5zi+16}\right| = \frac{\sqrt{10}}{\sqrt{17}}.$$

$$34\left|\frac{5z-12}{5zi+16}\right|^2 = 34\times\frac{10}{17} = 20.$$

11. Let $f(x) = x^3 + x^2 f'(1) + 2xf''(2) + f'''(3)\ \forall x \in R$ then the value of $f'(5)$ is

(1) $\frac{109}{5}$ (2) $\frac{117}{5}$

(3) $\frac{119}{5}$ (4) $\frac{118}{5}$

Answer (2)

Sol. Let $f'(1) = a$

$f'(2) = b$

$f'(3) = c$

$f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

$f'(1) = a = 3 + 2a + b \Rightarrow a + b = 3 \ldots(1)$

$f''(x) = 6x + 2a$

$\Rightarrow f''(2) = 12 + 2a = \frac{b}{2} \Rightarrow 4a - b = -24$

$\Rightarrow f''(x) = 6$

$\Rightarrow f''(3) = c = 6$

$\Rightarrow a = \frac{-27}{5}, b = \frac{12}{5}$

$f'(5) = 75 + 10a + b$

$= 75 - 54 + \frac{12}{5}$

$= 21 + \frac{12}{5} = \frac{117}{5}$

12.

13.

14.

15.

16.

17.

18.

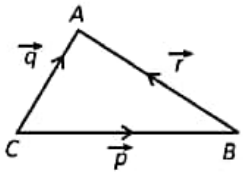
19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If three vectors are given as shown.

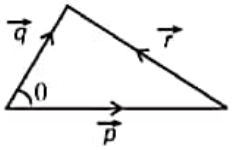


If angle between vectors $\vec{p}$ and $\vec{q}$ is θ where $\cos\theta = \frac{1}{\sqrt{3}}$ and $|\vec{p}| = 2\sqrt{3}, |\vec{q}| = 2$.

Then the value of $|\vec{p}\times(\vec{q} - 3\vec{r})|^2 - 3|\vec{r}|^2$ is

Answer (104)

Sol.



$\vec{p} + \vec{r} - \vec{q} = 0 \Rightarrow \vec{r} = \vec{q} - \vec{p}$

$$\cos\theta = \frac{|\vec{p}|^2 + |\vec{q}|^2 - |\vec{r}|^2}{2|\vec{p}||\vec{q}|} = \frac{12 + 4 - |\vec{r}|^2}{2\times 2\times 2\sqrt{3}}$$

$$\frac{16 - |\vec{r}|^2}{8\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow |\vec{r}|^2 = 8$$

$$|\vec{p}\times(\vec{q} - 3(\vec{q} - \vec{p}))|^2 - 3|\vec{r}|^2$$

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$$=\left|-2\vec{p}\times\vec{q}\right|^2-3\left|\vec{r}\right|^2$$

$$=4\times\left|\vec{p}\right|\left|\vec{q}\right|^2\sin^2\theta-3\times8$$

$$=4\times12\times4\times\left(\frac{2}{3}\right)-24$$

$$=128-24=104$$

22. The largest value of $n\in N$ such that 7^n divides (101)! is _____.

Answer (16)

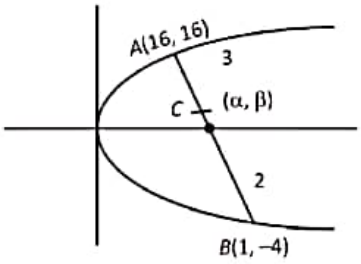
Sol. Exponent of 7 in $101!=\left[\frac{101}{7}\right]+\left[\frac{101}{7^2}\right]+....$ (where [·] is G.I.F.)

$=14+2=16$

23. Let $y^2=16x$, from point (16, 16) a focal chord is passing. Point (α, β) divides the focal chord in ratio 2 : 3, then minimum value of $\alpha+\beta$ is

Answer (11)

Sol. $y^2=16x$



$\therefore$ AB is focal chord $\Rightarrow t_1t_2=-1$

$t_1=2$

$t_2=-\frac{1}{2}$

$\Rightarrow B(1,-4)$

C can be $\left(\frac{2+48}{5},\frac{-8+48}{5}\right)$ or $\left(\frac{3+32}{5},\frac{-12+32}{5}\right)$

(10, 8) or (7, 4)

$\Rightarrow (\alpha+\beta)_{min}=11$

24. The minimum value of $(\sin^{-1}x)^2+(\cos^{-1}x)^2$ in $x\in\left[-\frac{\sqrt{3}}{2},\frac{1}{\sqrt{2}}\right]$ is $\frac{a\pi^2}{b}$, then $a+b$ is

Answer (9)

Sol. $(\sin^{-1})^2+(\cos^{-1}x)^2$

$$E=(\sin^{-1}x+\cos^2x)^2-2(\sin^{-1}x)(\cos^{-1}x)$$

$$=\left(\frac{\pi}{2}\right)^2-2(\sin^{-1}x)\left(\frac{\pi}{2}-\sin^{-1}x\right)$$

$$=\frac{\pi^2}{4}-(\sin^{-1}x)\pi+2(\sin^{-1}x)^2$$

$$E=2\left\{\left(\sin^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^2}{8}-\frac{\pi^2}{16}\right\}$$

$$=2\left\{\left(\sin^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^2}{16}\right\}$$

For min $\sin^{-1}x=\frac{\pi}{4}$ *i.e.* $x=\frac{1}{\sqrt{2}}$

$$E_{min}=\frac{\pi^2}{8}$$

25. If $A=\begin{bmatrix}3&-4\\1&-1\end{bmatrix}$, $B=\begin{bmatrix}23&49\\45&21\end{bmatrix}$ and if $\left(A^{15}+B\right)\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}21\\36\end{bmatrix}$ then $(x+2y)$ is equal to

Answer (7)

Sol. $A=\begin{bmatrix}3&-4\\1&-1\end{bmatrix}, B=\begin{bmatrix}23&49\\45&21\end{bmatrix}$

Charactertic equation of A,

$(3-\lambda)(-1-\lambda)+4=0$

$(\lambda-3)(\lambda+1)+4=\lambda^2-2\lambda+=0$

$\Rightarrow A^2-2A+I=0$

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$A^2 = 2A - I$, $A^3 = 2A^2 - A = 2(2A - I) - A = 3A - 2I$

$A^4 = 4A^2 + I - 4A = 4(2A - I) - 4A + I$

$= 4(2A - I) - 3A = 5A - 4I$

$\left(A^5\right)^3 = (5A - 4I)^3$

$= 125A^3 - 3 \times 25A^2(4) + 3(5A)(4^2) - 64I$

$= 125(3A - 2I) - 300(2A - I) + 240A - 64I$

$= A(375 - 600 + 240) + I(-250 + 300 - 64)$

$= 15A - 14I$

$\Rightarrow A^{15} + B = 15A - 14I + B$

$= 4A - 3I$

$A^5 = 4A^2 - 3A$

$= \begin{bmatrix} 45 & -60 \\ 15 & -15 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} + \begin{bmatrix} 23 & 49 \\ 45 & 21 \end{bmatrix}$

$= \begin{bmatrix} 54 & -11 \\ 60 & -8 \end{bmatrix} \Rightarrow \left(A^{15} + B\right) \begin{bmatrix} x \\ y \end{bmatrix}$

$\Rightarrow \quad 54x - 11y = 21 \qquad \Rightarrow \quad x = 1, y = 3$

$60x - 8y = 36$

$\Rightarrow \quad x + 2y = 7$

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