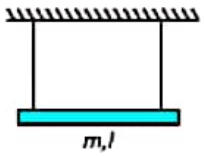
Physics

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A rod of mass m and length l is attached to two ideal strings. Find tension in left string just after right string is cut.

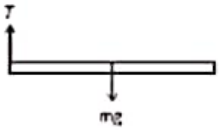


(1) $\frac{mg}{2}$ (2) $\frac{mg}{4}$

(3) $\frac{2}{3}mg$ (4) $\frac{mg}{5}$

Answer (2)

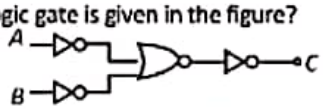
Sol. $\alpha = \frac{\frac{mg\frac{l}{2}}{\frac{ml^2}{3}} - \frac{3g}{2l}}{}$



$a_{com} = \frac{3g}{4}$

$T = \frac{mg}{4}$

2. Which logic gate is given in the figure?



(1) XOR (2) NOR

(3) NAND (4) OR

Answer (3)

Sol. $\overline{\overline{A}+\overline{B}} = \overline{AB}$

3. Find dimensions of $\frac{A}{B}$ if $\left(P + \frac{At^2}{B}\right) + \frac{1}{2}\rho V^2 = \text{constant}$ where $P \rightarrow$ pressure, $\rho \rightarrow$ density, $V \rightarrow$ speed.

(1) ML^1T^{-4} (2) $ML^{-1}T^{-4}$

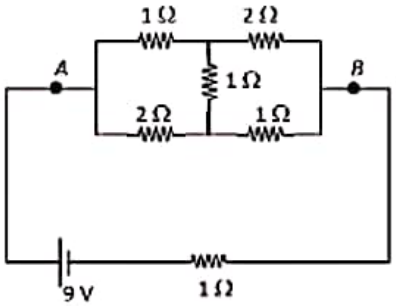
(3) ML^2T^{-4} (4) $ML^{-1}T^{-2}$

Answer (2)

Sol. $\left[\frac{At^2}{B}\right] = [P] = ML^{-1}T^{-2}$

$\left[\frac{A}{B}\right] = ML^{-1}T^{-4}$

4. Find the heat produced in external circuit (AB) in one minute.

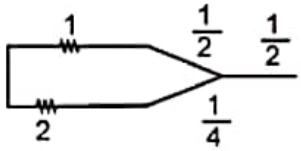


(1) 1181.25 J (2) 1311.25 J

(3) 1207.50 J (4) 1410.50 J

Answer (1)

Sol. You can use Kirchoff's law or star-delta



$R_{AB} = \frac{3 \times \frac{9}{4}}{\frac{3}{2}+\frac{9}{4}} + \frac{1}{2} = 1.4\ \Omega ; P = i^2R$

5. An α-particle having kinetic energy 7.7 MeV is approaching fixed gold nucleus (atomic number is 79). Find distance of closest approach.

(1) 1.72 nm (2) 6.2 nm

(3) 16.8 nm (4) 0.2 nm

Answer (1)

Sol. $_2^4He \xrightarrow{v} ..... v + 79e$

$$\frac{1}{2}mv^2 = \frac{K(2e)(79e)}{r^2}$$

$$7.7\times10^5\times1.6\times10^{-19}J = \frac{9\times10^9\times158\times(1.6\times10^{-19})^2}{r^2}$$

$$r^2 = \frac{2275.2\times10^{-12}}{7.7\times10^5}$$

$r = 17.2\times10^{-8}$

$r = 17$ nm

296×10^{-16}

6. An air filled capacitor of capacitance C is filled with dielectric ($k = 3$) of width $d/3$, where d is separation between plates. The new capacitance is

(1) $\frac{9}{5}C$ (2) $\frac{5}{4}C$

(3) $\frac{4}{3}C$ (4) $\frac{9}{7}C$

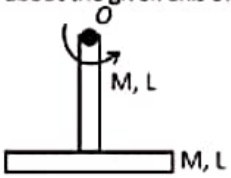
Answer (4)

Sol. $C = \dfrac{\epsilon_0 A}{\frac{d_1}{k_1} + \frac{d_2}{k_2}}$

$$\frac{\epsilon_0 A}{\frac{d}{3\times3} + \frac{2d}{3}}$$

$$= \frac{9\epsilon_0 A}{d + 6d} = \frac{9\epsilon_0 A}{7d}$$

7. Find the moment of inertia of system formed using two identical rods about the given axis of rotation as shown



(1) $\frac{17}{12}ML^2$ (2) $\frac{13}{12}ML^2$

(3) $\frac{2ML^2}{3}$ (4) $\frac{3ML^2}{4}$

Answer (1)

Sol. For vertical rod about O $I_{10} = \dfrac{ML^2}{3}$

For horizontal rod about O $I_{20} = \dfrac{ML^2}{12} + ML^2 = \dfrac{13}{12}ML^2$

$$I_{O_{sys}} = I_{10} + I_{20} = \frac{17}{12}ML^2$$

8. If electric field of EM wave is given by $60[\sin(3\times10^{14}t) + \sin(12\times10^{14}t))]$ at $x = 0$ falls on a photo sensitive material having work function 2.8 eV. Find the maximum kinetic energy (M eV) of ejected electrons.

(1) 2.52 eV (2) 2.16 eV

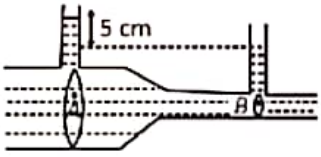
(3) 2.00 eV (4) 2.34 eV

Answer (2)

Sol. $\dfrac{h\nu}{c} = 4.963$ eV

$KE_{max} = 4.963 - 2.8 = 2.163$ eV

9. Find volume flow rate in the venturi meter given below in which water is flowing.



[cross section area at A & B is A & a, $\frac{A}{a} = 2.4A = \sqrt{3}$ m^2, $P = 1000$ kg/m^3.]

(1) 1 (2) $\sqrt{3}$

(3) $2\sqrt{3}$ (4) $\sqrt{2}$

Answer (1)

Sol. $P_A+\frac{1}{2}PV_A^2 = P_B+\frac{1}{2}PV_B^2$

$P_A - P_B = \frac{1}{2}P\left(V_B^2 - V_A^2\right)$

$\Rightarrow \quad V_B^2 - V_A^2 = 1$

and $AV_A = aV_B$

$\Rightarrow \quad 3V_A^2 = 1 \quad \Rightarrow \quad V_A = \frac{1}{\sqrt{3}}$

10. An ideal solenoid is kept with its axis vertical. Current I_0 is flowing in the solenoid. A charge Q is thrown downward inside the solenoid it acceleration of the charge particle is a then

(1) $a > g$ (2) $a = g$

(3) $a < g$ (4) $a = 0$

Answer (2)

Sol. $\vec{V} \| \vec{B} \Rightarrow F_m = 0$

$a = g$

11. Wave propagates whose electric field is given by $\vec{E} = 69\sin(\omega t - kx)\hat{j}$ find the direction of magnetic field

(1) $\hat{k}$ (2) $-\hat{k}$

(3) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (4) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$

Answer (1)

Sol. $\hat{E} \Rightarrow \hat{j}, \hat{C} \Rightarrow \hat{i}$

$\vec{E} \times \vec{B} = \vec{C}$

$B = \hat{k}$

12. Two rods of equal length of 60 cm each are joined together end to end. Coefficient of linear expansions of the rods are $24 \times 10^{-6}\ C^{-1}$ and $1.2 \times 10^{-5}\ C^{-1}$. Their temperatures are same and equal to 30°C which is increased to 100°C. Find final length of the combination (in cm).

(1) 120.1321 (2) 120.1123

(3) 120.1512 (4) 120.1084

Answer (3)

Sol. $\Delta\ell_1\ell \quad +_2 = 60\ (3.6 \times 10^{-5} \times 70)$

$\Rightarrow \quad 15.12 \times 10^{-2}\ \text{cm} = 0.1512\ \text{cm}$

$\ell_f = 120 + 0.1512 = 120.1512\ \text{cm}$

13. Find change in internal energy of gas if its temperature changes by 10K. Number of moles of gas is 10, C_P (specific heat at constant pressure of the gas is 7 cal/K-mol) and R (gas constant) = 2 cal/K.

(1) 500 cal (2) 1000 cal

(3) 250 cal (4) 100 cal

Answer (1)

Sol. CP – CV = R = 2

$C_V = 5$

$\Delta V = nC_V\Delta T = 10 \times 5 \times 10 = 500\ \text{cal}$

14. Two mechanical wave on strings of equal length (L) tension (T) having linear mass density $\frac{\mu_1}{\mu_2} = \frac{1}{2}$. Find the ratio of time taken for a wave pulse to travel from one end to the other in both strings. (ignore gravity)

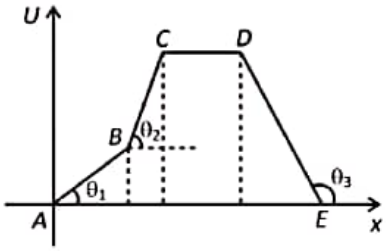
(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\sqrt{2}$ (4) 2

Answer (2)

Sol. $C = \sqrt{\frac{T}{\mu}}$ and $t = \frac{L}{C}$ $t \propto \sqrt{\mu} \Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{1}{2}}$

15. A curve is given between potential energy of a particle and its position on *x*-axis.



Given: $\tan\theta_1 = 1$, $\tan\theta_2 = 3$, $\tan\theta_2 = \frac{-1}{2}$

If F_{AB} be force acting on the particle during *A* to *B* similarly F_{BC}, F_{CD} and F_{DE} are the forces during *B* to *C*, *C* to *D* and *D* to *E* respectively. Arrange magnitudes of these forces in decreasing order

(1) $F_{BC} > F_{AB} > F_{CD} > F_{DE}$ (2) $F_{BC} > F_{AB} > F_{DE} > F_{CD}$

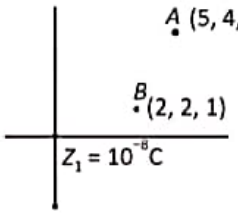
(3) $F_{AB} > F_{BC} > F_{DE} > F_{CD}$ (4) $F_{BC} > F_{DE} > F_{AB} > F_{CD}$

Answer (2)

Sol. $F = -\frac{dU}{dx}$

$\therefore$ Higher the slope greater the force.

16. Find out work done in moving a 2μC. Choose from *A* to *B*.



(1) 6 μJ (2) 120 mJ

(3) 34.3 μJ (4) 24.2 μJ

Answer (3)

Sol. $w = U_2 - U_1 = 9\times10^9 \times10^{-8}\times2\times10^{-6}\left(\frac{1}{3}-\frac{1}{7}\right)$

= 34.3 μJ

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. 21. A satellite is revolving around a planet in orbit radius of 1.5 R. Additional minimum energy required to transfer the satellite to new orbit radius of 3R is (m and M are mass of satellite & planet) $\frac{GMm}{\lambda R}$ then *X* is _____.

Answer (6)

Sol. $M.E = \frac{-GMm}{2a}$

$W = \Delta M = M_f - M_i$

$= \frac{-GMm}{2(3R)} - \left(-\frac{GMm}{2(1.5R)}\right)$

$= +\frac{GMm}{R}\left\{-\frac{1}{6}+\frac{1}{3}\right\}$

$= \frac{GMm}{6R}$

22. There are two springs of spring constants $k_1 = (20 \pm 0.2)$ N/m and $k_2 = (30 \pm 0.3)$ N/m. If they are connected in parallel then percentage error in equivalent spring constant of combination is _____%.

Answer (1)

Sol. $\Delta k = \Delta k_1 + \Delta k_2 = 0.5$

Keq = 50 N/m

% error = $\frac{0.5}{50}\times100 = 1$

23. In a *YDSE* set up, a slab of width t is inserted in front of one of slit. The interference pattern shifts by 0.2 cm on the screen. If the refractive index of slab is 1.5 than t is N μm (screen distance 50 cm and slits separation 1 mm) then N is _____

Answer (8)

Sol. Path difference by shift is neutralised from path difference by slab $\frac{dy}{D} = (\mu - 1)t$.

$$\frac{10^{-3}}{0.5} \times 0.2 \times 10^{-2} = \frac{1}{2} \times t$$

$$10^{-3} \times \frac{2}{5} \times 2 \times 10^{-2} = t$$

$$10^{-5} \times \frac{4}{5} = t$$

$0.8 \times 10^{-5} = 8\ \mu m = t$

24. A particle of mass 1 kg, initially resting at origin, starts moving under the influence of a force $\vec{F} = 4t^3\hat{i} - 3t^2\hat{j}$. If the speed of the particle at $t = 1$ is $\sqrt{\alpha}$, then value of α is

Answer (2)

Sol. $v_x = 4t^3 = \frac{dv_x}{dt}$

$v_x = 1$ m/s

$v_y = -3t^2$

$v_y = 1$ m/s

$v = \sqrt{2}$ m/s

25. Focal length of objective lens and eyepiece lens are 1.25 cm and 5 cm and tube length is 26 cm. Find magnification of compound microscope in normal adjustment.

Answer (104)

Sol. $M = \frac{L}{f_0} \cdot \frac{D}{f_e}$

$= \frac{26}{1.25} \times \frac{25}{5}$

$M = 104$

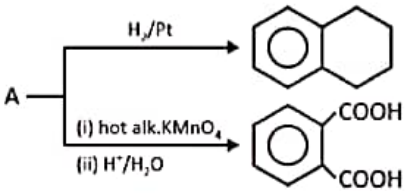
CHEMISTRY

SECTION - A

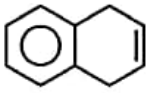
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

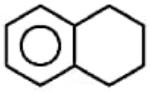
Choose the correct answer :

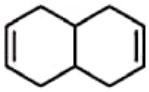
1. Consider the following reaction :

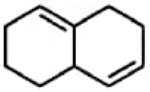


Then 'A' will be

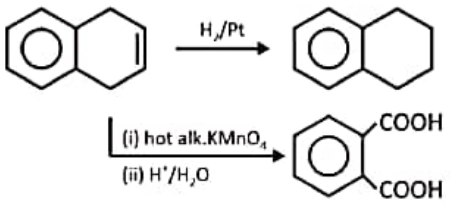
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

2. In Sulphur estimation, 0.7 g of an organic compound gives 1 g of $BaSO_4$ by Carius method. What is the % of 'S' in compound?

(1) 19.61

(2) 23.85

(3) 27.93

(4) 14.57

Answer (1)

Sol. $$\% \text{ of } S = \frac{\frac{1}{233} \times 32}{0.7} \times 100 = 19.61\%$$

3. Which of the following is the correct order with respect to the property indicated?

(1) Cl > F (Ionisation energy)

(2) $K_2O > Na_2O > Al_2O_3$ (Basic nature)

(3) K > Na > Al > Mg (Metallic character)

(4) None of these

Answer (2)

Sol. F > Cl : First ionisation energy (due to small size of F)

K > Na > Mg > Al : Metallic character (It decreases from Left to Right across the period and increases from Top to Bottom.

4. Given below are two statements.

Statement I : Arginine and Tryptophan are essential amino acids.

Statement II : Glycine does not have any chiral carbon.

In the light of the above statements, which is the correct option.

(1) Both statement-I and statement-II are correct

(2) Both statement-I and statement-II are incorrect

(3) Statement-I is correct and statement-II is incorrect

(4) Statement-I is incorrect and statement-II is correct

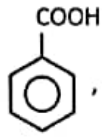
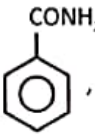
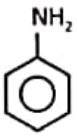
Answer (1)

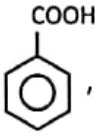
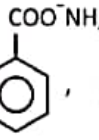
Sol. Arginine are Tryptophan both are essential amino acids. Glycine does not contain any chiral centre.

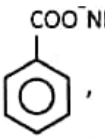
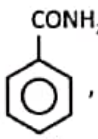
5. Observe the following reaction sequence:

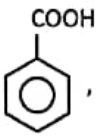
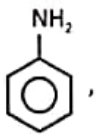
$$(P) \xrightarrow[\Delta]{NH_3} (Q) \xrightarrow{Br_2, KOH} (R)$$

Which of the following is the correct structure for P, Q and R?

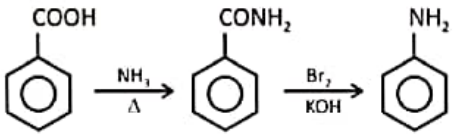
(1) P =  , Q =  , R = 

(2) P =  , Q =  , R = 

(3) P =  , Q =  , R = 

(4) P =  , Q =  , R = 

Answer (1)

Sol. 

6. In the following reaction,

$$MnO_4^{2-} \xrightarrow{H^+}$$

Manganate ion undergoes disproportionation in acidic medium to form

(1) MnO_2, MnO_4^- (2) MnO, MnO_2

(3) MnO_2, Mn_2O_3 (4) MnO_4^-, MnO

Answer (1)

Sol. MnO_4^{2-} disproportionates in acidic solution to gives MnO_4^- and MnO_2

$$3MnO_4^{2-} + 4H^+ \longrightarrow 2MnO_4^- + MnO_2 + 2H_2O$$

7. 80 mL of an organic compound is mixed with 264 mL O_2 and ignited. It gives 224 mL of gaseous mixture at NTP. After passing through KOH 64 mL of gas remains. The organic compound is

(1) C_2H_4 (2) C_2H_2

(3) C_4H_{10} (4) C_3H_6

Answer (2)

Sol. $$C_xH_y(g) + \left(x+\frac{y}{4}\right)O_2(g) \rightarrow xCO_2(g) + \frac{y}{2}H_2O(l)$$

t = 0 80 mL 264 mL

$$\left(V_{CO_2} + V_{O_2}\right)_{\text{after reaction}} = 224 \text{ mL}$$

After passing through KOH, 64 mL gas left

$$\left(V_{O_2}\right)_{\text{left}} = 64 \text{ mL}$$

$$\left(V_{O_2}\right)_{\text{used}} = 200 \text{ mL}$$

$$\left(V_{CO_2}\right)_{\text{formed}} = 224 - 64 = 160 \text{ mL}$$

1 mL $C_xH_y \rightarrow$ x mL CO_2

80 mL $\rightarrow$ 160 mL CO_2

$\boxed{x=2}$

$$\frac{V_{C_xH_y}}{1} = \frac{\left(V_{O_2}\right)_{\text{used}}}{x+\frac{y}{4}}$$

$$80 = \frac{200}{\left(2+\frac{y}{4}\right)}$$

160 + 20y = 200

20y = 40

$\boxed{y=2}$ formula C_2H_2

8. Consider the following reaction

$Ca + 2HCl \rightarrow CaCl_2 + H_2$

We have 14 g Ca reacts with excess of HCl. Choose the incorrect option.

(1) Mass of $CaCl_2$ produced is 38.85 g

(2) Mole of H_2 produced is 0.35 mol

(3) Volume of H_2 produced at STP is 7.84 L

(4) Mass of $CaCl_2$ produced is 3.885 g

Answer (4)

Sol. $Ca + 2HCl \rightarrow CaCl_2 + H_2$

$\frac{14}{40} = 0.35\,mol$

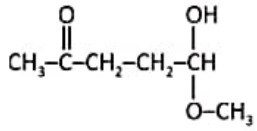
$n_{CaCl_2} = 0.35$

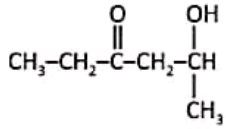
$W_{CaCl_2} = 38.85\,g$

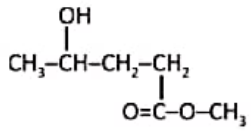
$n_{H_2} = 0.35$

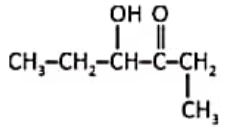
V_{H_2} at STP = 0.35 × 22.4 = 7.84 L

9. $C_6H_{12}O_3$ gives positive iodoform test on hydrolysis with dil. Acid. The hydrolysis product formed gives Tollens' and iodoform test both. Find structure of $C_6H_{12}O_3$.

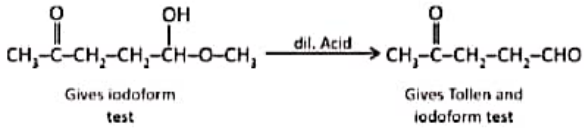
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

10. Given below are two statements :

Statement I : All the pairs of molecules (PbO, PbO_2); (SnO, SnO_2) and (GeO, GeO_2) contain amphoteric oxides.

Statement II : $AlCl_3$, BH_3, BeH_2 and NO_2 all have incomplete octet.

In the light of the above statements, choose the correct option.

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

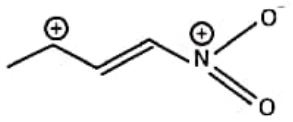
(3) Statement I is correct but Statement II is incorrect

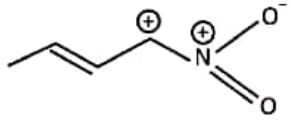
(4) Statement I is incorrect but Statement II is correct

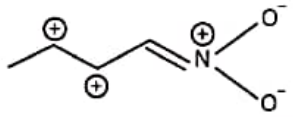
Answer (4)

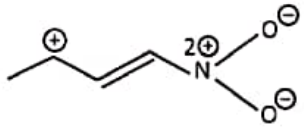
Sol.
- SnO, SnO_2, PbO, PbO_2 are amphoteric oxides.
- GeO_2 is acidic.
- GeO is distinctly acidic.

11. Which of the following resonating structure is the most stable?

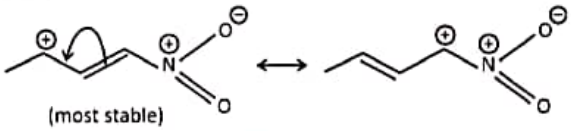
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

12. Consider the following statements.

(A) Propanal and Propanone are functional isomers

(B) Ethoxyethane and methoxypropane are metamers

(C) But-2-ene shows optical isomerism

(D) But-1-ene and But-2-ene are functional isomers

(E) Pentane and 2, 2-dimethylpropane are chain isomers

The correct statements are

(1) A, B, D only

(2) B, C, D only

(3) A, B, E only

(4) A, B, D, E only

Answer (3)

Sol.

- But-2-ene doesn't show optical isomerism as it contain plane of symmetry and has no chiral centre also.
- But-1-ene and But-2-ene are position isomers.

13. Given below are two statements

Statement I: When electric discharge is put on hydrogen, it emits discrete frequency in electromagnetic spectrum.

Statement II: Frequency of He^+ ion of 2nd line of Balmer series is equal to first line of Lyman series.

(1) Both statement I and statement II are correct

(2) Both statement I and statement II are incorrect

(3) Statement I is correct and statement II is incorrect

(4) Statement I is incorrect and statement II is correct

Answer (1)

Sol. For He^+ ion

$$\nu \propto z^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right) \propto 2^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right)$$

$$\propto \frac{3}{4}$$

For H atom

$$\nu \propto 1^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right) \propto \frac{3}{4}$$

$\therefore$ frequency is same.

14. Which of the following compound is paramagnetic in nature?

(1) $[Ni(CO)_4]$

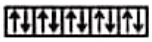
(2) $[Ni(CN)_4]^{2-}$

(3) $[NiCl_4]^{2-}$

(4) $[Co(H_2O)]^{3+}$

Answer (3)

Sol.

$Ni(CO)_4$ $\quad 3d^8 4s^2$  sp^3

n = 0

$Ni(CN)_4^{2-}$ $\quad 3d^8$  dsp^2

$\because$ CN^- is SFL

n = 0

$NiCl_4^{2-}$ $\quad 3d^8$  sp^3

n = 2

Paramagnetic

$[Co(H_2O)_6]^{3+}$ $\quad 3d^6$  d^2sp^3

n = 0

H_2O with Co^{3+} is SFL

15. Observe the following reaction sequence :

$$\underset{(A)}{C_4H_8} \xrightarrow{HCl} \underset{(B)}{C_4H_9Cl} \xrightarrow{NH_3} \underset{\text{(Optically active)}}{C_4H_9NH_2}$$

(C)

$\xrightarrow{NaNO_2 + HCl}$

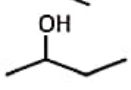
C_4H_9OH

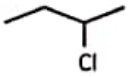
Which of the following is correct structure of A, B and C?

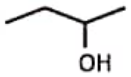
(1) (A) =  , (B) = 

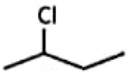
(C) = 

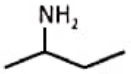
(2) (A) =  , (B) = 

(C) = 

(3) (A) =  , (B) = 

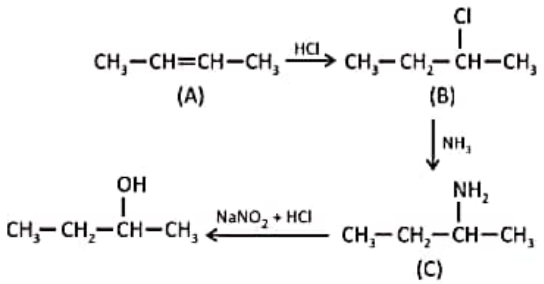
(C) = 

(4) (A) =  , (B) = 

(C) = 

Answer (4)

Sol.



16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. For two chemical reactions A and B, if the difference between their activation energy is 20 kJ at 300 K (R = 8.3 J K^{-1} mol^{-1}). Determine $\ln\frac{K_2}{K_1}$.

Answer (8)

Sol. For reaction A,

$$k_1 = Ae^{\frac{-E_{a_1}}{RT}}$$

For reaction B,

$$k_2 = Ae^{\frac{-E_{a_2}}{RT}}$$

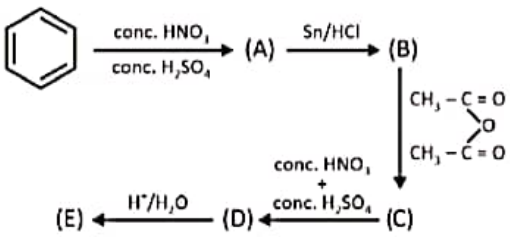
$$\frac{k_2}{k_1} = \frac{e^{\frac{-E_{a_2}}{RT}}}{e^{\frac{-E_{a_1}}{RT}}}$$

$$\frac{k_2}{k_1} = e^{\frac{-E_{a_2}+E_{a_1}}{RT}}$$

$$\ln\frac{k_2}{k_1} = \frac{-E_{a_2}+E_{a_1}}{RT}$$

$$\ln\frac{k_2}{k_1} = \frac{20000}{8.3\times300}$$

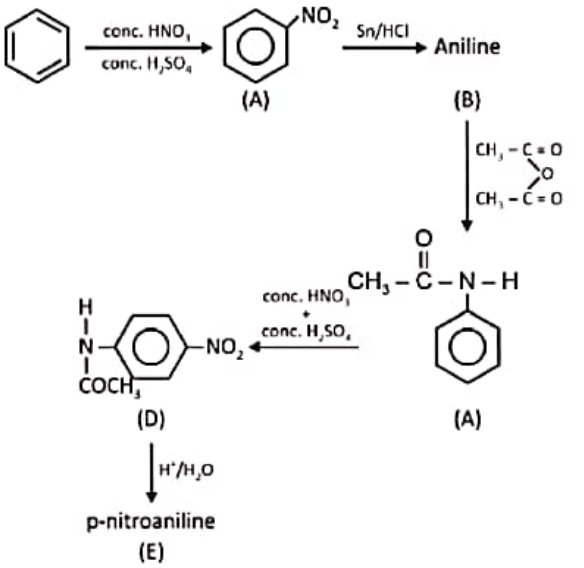
$$\ln\frac{k_2}{k_1} \approx 8$$

22. 

% of N in compound E is ____

Answer (20)

Sol.



$$\% \text{ of } N = \frac{14 \times 2 \times 100}{138} = 20.28 \approx 20$$

23. 1 g of AB_2 is dissolved in 50 g solvent such that ΔT_f = 0.689. When 1 g AB is dissolved in 50 g of same solvent, ΔT_f is 1.176. Find molar mass of AB_2. K_f = 5 K kg/mol.

AB_2 and AB are non electrolyte. (Report to nearest integer)

Answer (145 g)

Sol. Let 'a' and 'b' are atomic weight of 'A' and 'B' respectively

$$0.689 = 5\left[\frac{1}{a+2b} \times \frac{1000}{50}\right] \quad ...(1)$$

$$1.176 = 5\left[\frac{1}{a+b} \times \frac{1000}{50}\right] \quad ...(2)$$

$$\frac{0.689}{1.176} = \frac{a+b}{a+2b} = \frac{1}{1.7}$$

$\Rightarrow$ 1.7a + 1.7b = a + 2b

0.7a = 0.3b

$$b = \frac{7}{3}a$$

$$\text{Now, } 1.176 = \left[\frac{1}{a+\frac{7}{3}a} \times 20\right] \times 5 = \frac{300}{10a}$$

$$\Rightarrow a = \frac{30}{1.176} = 25.51$$

$$b = \frac{7}{3}a = 59.52 \text{ g}$$

$$M_{AB_2} = 25.51 + 2 \times 59.52 = 144.55 \text{ g}$$

24. Out of the following, how many compounds have tetrahedral geometry?

$\overset{+}{N}H_4$, XeF_4, $[NiCl_4]^{2-}$, $[PtCl_4]^{2-}$, $[Cu(NH_3)_4]^{2+}$, BF_3 and $[Ni(CO)_4]$

Answer (3)

Sol.

Species		Geometry
$\overset{+}{N}H_4$	$\Rightarrow$	Tetrahedral
XeF_4	$\Rightarrow$	Octahedral
$[NiCl_4]^{2-}$	$\Rightarrow$	Tetrahedral
$[PtCl_4]^{2-}$	$\Rightarrow$	Square planar
$[Cu(NH_3)_4]^{2+}$	$\Rightarrow$	Square planar
BF_3	$\Rightarrow$	Triangular Planar
$[Ni(CO)_4]$	$\Rightarrow$	Tetrahedral

25.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of $\text{cosec}10° - \sqrt{3}\sec 10°$

(1) 1 (2) 2

(3) 4 (4) 5

Answer (3)

Sol. $\frac{1}{\sin 10} - \frac{\sqrt{3}}{\cos 10}$

$= \frac{1}{2}\left[\frac{\cos 10° - \sqrt{3}\sin 10°}{\frac{1}{2}\sin 10°\cos 10°}\right]$

$= 2\times 2\frac{[\sin 30°\cos 10° - \cos 30°\sin 10°]}{\sin 20°} = 4$

2. If $A = \begin{bmatrix}\alpha & 2\\ 1 & 2\end{bmatrix}$, $B = \begin{bmatrix}1 & 1\\ \beta & 1\end{bmatrix}$ and $A^2 - 4A + 2I = 0$, $B^2 - 2B + I = 0$, then $|\text{adj}\,(A^3 - B^3)|$ is equal to

(1) 11

(2) 7

(3) −11

(4) 121

Answer (1)

Sol. $\begin{vmatrix}\alpha-\lambda & 2\\ 1 & 2-\lambda\end{vmatrix} = 0$

$(\lambda - \alpha)(\lambda - 2) - 2 = 0$

$\lambda^2 + \lambda(-2 - \alpha) + 2\alpha - 2 = 0$

$-2 - \alpha = -4 \Rightarrow \alpha = 2$

Similarly,

$\begin{vmatrix}1-\mu & 1\\ \beta & 1-\mu\end{vmatrix} = 0$

$\mu^2 - 2\mu + 1 - \beta = 0$

$1 - \beta = 1 \Rightarrow \beta = 0$

$\Rightarrow A^3 = 4A^2 - 2A$

$= 4(4A - 2I) - 2A$

$= 14A - 8I$

$B^3 = 2B^2 - B$

$= 2(2B - I) - B$

$= 3B - 2I$

$A^3 - B^3 = 14A - 3B - 6I$

$= 14\begin{vmatrix}2 & 2\\ 1 & 2\end{vmatrix} - 3\begin{vmatrix}1 & 1\\ 0 & 1\end{vmatrix} - 6\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} = 0$

$= \begin{vmatrix}19 & 25\\ 14 & 19\end{vmatrix} \Rightarrow \det(A^3 - B^3) = 11$

$= |\text{adj}(A^3 - B^3)| = |A^3 - B^3| = 11$

3. If $y = y(x)$ and $(1 + x^2)dy + (1 - \tan^{-1}x)dx = 0$ and $y(0) = 1$ then $y(1)$ is

(1) $\frac{\pi^2}{32} + \frac{\pi}{4} + 1$

(2) $\frac{\pi^2}{32} - \frac{\pi}{4} + 1$

(3) $\frac{\pi^2}{32} - \frac{\pi}{2} - 1$

(4) $\frac{\pi^2}{32} - \frac{\pi}{2} + 1$

Answer (2)

Sol. $(1 + x^2)dy + (1 - \tan^{-1}x)dx = 0$

$dy + \frac{1 - \tan^{-1}x}{1 + x^2}dx = 0$

$$\int dy+\int\frac{dx}{1+x^2}-\int\frac{\tan^{-1}x}{1+x^2}dx=0$$

Or $y+\tan^{-1}x-I=0$, where

$$I=\int\frac{\tan^{-1}x}{1+x^2}dx$$

Let $\tan^{-1}x=t$

$$\frac{1}{1+x^2}dx=dt$$

$$\int tds\Rightarrow\frac{t^2}{2}=\frac{(\tan^{-1}x)^2}{2}$$

$$\Rightarrow\ y+\tan^{-1}x-\frac{(\tan^{-1}x)^2}{2}+c=0$$

$$\therefore\ y(0)=1\qquad 1+c=0\quad\Rightarrow c=-1$$

$$\Rightarrow\ y=\frac{(\tan^{-1}x)^2}{2}-\tan^{-1}x+1$$

Now $y(1)=\frac{1}{2}\left(\frac{\pi}{4}\right)^2-\frac{\pi}{4}+1$

4. The value of

$$\int_0^{\pi/2}|\sin x+\sin 2x+\sin 3x|dx \text{ is}$$

(1) $\frac{8}{3}$

(2) $\frac{7}{3}$

(3) $\frac{2}{3}$

(4) 3

Answer (2)

Sol. $\because \sin x+\sin 3x=\sin(2x-x)+\sin(2x+x)$

$=2\sin 2x\cos x$

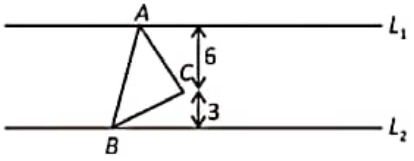
$\therefore \sin 3x+\sin 2x+\sin x=\sin 2x\,(2\cos x+1)$

$$\therefore\int_0^{\pi/2}|\sin 2x(2\cos x+1)|dx$$

$$=\int_0^{\pi/2}(\sin 2x)dx+\int_0^{\pi/2}4\cos^2 x\sin x dx$$

$$=\frac{-\cos 2x}{2}\Big|_0^{\pi/2}+\left(\frac{-4}{3}(\cos^3 x)\right)\Big|_0^{\pi/2}=\frac{7}{3}$$

5. If L_1 and L_2 are two parallel lines and ΔABC is an equilateral triangle then area of triangle ABC is

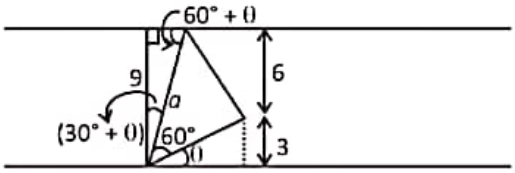


(1) $7\sqrt{3}$ (2) $4\sqrt{3}$

(3) $21\sqrt{3}$ (4) 84

Answer (3)

Sol. Let a be the side of ΔABC



$$\sin\theta=\frac{3}{a}$$

$$\sin(60^\circ+\theta)=\frac{9}{a}$$

$$\Rightarrow\ \frac{\sqrt{3}}{2}\times\sqrt{1-\frac{9}{a^2}}+\frac{3}{a}\times\frac{1}{2}=\frac{9}{a}$$

$$\Rightarrow\ \sqrt{3}\left(\sqrt{a^2-9}\right)+3=18$$

$$\Rightarrow\ 3(a^2-9)=15^2\qquad\Rightarrow a^2-9=15\times 5$$

$$\Rightarrow a^2=84$$

$$\text{Area}=\frac{\sqrt{3}}{4}a^2=21\sqrt{3}$$

6. Consider a set $S = \{a, b, c, d\}$. Then number of reflexive as well as symmetric relations from $S \to S$ are

(1) 1024 (2) 256

(3) 16 (4) 64

Answer (4)

Sol. Number of such relations:

From $A \to A$ such that

$n(A) = N$ is

$\Rightarrow 2^{\left(\frac{N^2-N}{2}\right)}$

for $N = 4 \Rightarrow 2^6 = 64$

Alter :

$$\begin{bmatrix} O & - & - & - \\ \Theta & O & - & - \\ \Theta & \Theta & O & - \\ \Theta & \Theta & \Theta & O \end{bmatrix} \Rightarrow \text{each } \Theta \text{ has two choices}$$

$\Rightarrow 2^6 = 64$

7. The locus of point of intersection of tangent drawn to the circle $(x - 2)^2 + (y - 3)^2 = 16$, which sub-stends an angle of 120° is

(1) $3x^2 + 3y^2 - 12x - 18y - 25 = 0$

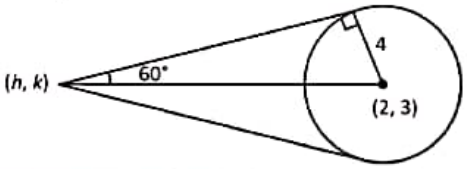
(2) $x^2 + y^2 - 12x - 18y - 25 = 0$

(3) $3x^2 + 3y^2 + 12x + 18y - 25 = 0$

(4) $x^2 + y^2 + 12x + 18y - 25 = 0$

Answer (1)

Sol.



$x^2 + y^2 - 14x - 6y - 3 = 0$

$$\tan 60^\circ = \frac{4}{\sqrt{h^2 + k^2 - 4h - 6k - 3}}$$

squaring both side

$3(h^2 + k^2 - 4h - 6k - 3) = 16$

To get locus replace h, k by x and y;

$3x^2 + 3y^2 - 12x - 18y - 9 - 16 = 0$

$3x^2 + 3y^2 - 12x - 18y - 25 = 0$

8. If $a_1, a_2, a_3, \ldots$ are the terms of an increasing geometric progression such that

$a_1 + a_3 + a_5 = 21,$

$a_1 a_3 a_5 = 64$

then $a_1 + a_2 + a_3$ is

(1) 5 (2) 7

(3) 10 (4) 15

Answer (2)

Sol. Let $a_3 = P, a_1 = \frac{P}{r^2}, a_5 = Pr^2$

$$\Rightarrow P + \frac{P}{r^2} + Pr^2 = 21$$

and $P \times \frac{P}{r^2} \times Pr^2 = 64 \Rightarrow P^3 = 64 = 4^3$

$\Rightarrow P = 4$

$$\Rightarrow 1 + \frac{1}{r^2} + r^2 = \frac{21}{4} \Rightarrow r^4 - \frac{17}{4} r^2 + 1 = 0$$

$$r^2 = \frac{\frac{17}{4} \pm \sqrt{\frac{289}{16} - 4}}{2} = \frac{\frac{17}{4} \pm \frac{15}{4}}{2} = 4, \frac{1}{4}$$

$$\Rightarrow r = \pm 2, \pm \frac{1}{2}$$

$$\Rightarrow a_1 + a_2 + a_3 = \frac{P}{r^2} + \frac{P}{r} + P$$

$$= 4\left(\frac{1}{4} + \frac{1}{2} + 1\right)$$

$$= 1 + 2 + 4 = 7$$

9. Ellipse $E : \frac{x^2}{36} + \frac{y^2}{16} = 1$, A hyperbola confocal with ellipse E and eccentricity of hyperbola is equal to 5. The length of latus rectum of hyperbola is, if principle axis of hyperbola is x-axis?

(1) $\frac{96}{\sqrt{5}}$ (2) $24\sqrt{5}$

(3) $18\sqrt{5}$ (4) $12\sqrt{5}$

Answer (1)

Sol. $E:\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$

$H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

For con-focal E and H

$\because\quad A^2-B^2=a^2+b^2$ (Given that $A^2=36$ and $B^2=16$)

$36-16=a^2+b^2$

$a^2+b^2=20 \qquad ...(1)$

$e_H=5$

$\Rightarrow \quad 25=1+\frac{b^2}{a^2}$

$\Rightarrow \quad 25a^2=20\Rightarrow a^2=\frac{4}{5}\Rightarrow a=\frac{2}{\sqrt{5}}$

Also $a^2+b^2=20$

$b^2=20-\frac{4}{5}\Rightarrow\frac{96}{5}$

$L(LR)=\ =2\times\frac{b^2}{a}=\frac{2\times\frac{96}{5}}{\frac{2}{\sqrt{5}}}=\frac{96}{\sqrt{5}}$

10. If the mean and variance of observations x, y, 12, 14, 4, 10, 2 is 8 and 16 respectively where $x > y$. Then, the value of $3x-y$ is

(1) 18 (2) 20

(3) 22 (4) 24

Answer (1)

Sol. $\frac{x+y+12+14+4+10+2}{7}=8$

$\Rightarrow x+y=14$

also var = 16

$\frac{x^2+y^2+144+196+16+100+4}{7}-(8)^2=16$

$\Rightarrow x^2+y^2=100$

By solving we get

$x=8, y=6$

$\Rightarrow 3x-y=18$

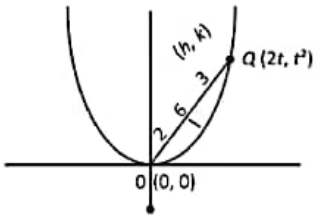
11. If O is the vertex of the of the parabola $x^2=4y$, Q is the point on parabola. If C is the locus of point which divides OQ in ratio 2:3. The equation of chord of C which bisected at point (1, 2).

(1) $5x+4y+3=0$ (2) $5x-4y-3=0$

(3) $5x-4y+3=0$ (4) $5x+4y-3=0$

Answer (C)

Sol.



$h=\frac{4t}{5}\ ,k=\frac{2t^2}{5}$

$\Rightarrow \frac{2t^2}{5}=t,\ \frac{2t^2}{5}=t^2$

$\frac{5k}{2}=\left(\frac{5h}{4}\right)^2$

Replace (h, k) with (x, y)

$\frac{5y}{2}=\frac{25x^2}{16}=$

$C:\ \boxed{8y=5x^2}$

chord with given middle point

$T=S_1$

$5xx_1-4(y+y_1)=5x_1^2-8y_1$

$5x-4(y+2)=5-16$

$5x-4y-8=-11$

$5x-4y+3=0$

12. The value of $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\frac{\pi+4x^{11}}{1-\sin\left(|x|+\frac{\pi}{6}\right)}dx$

(1) 3π (2) 4π

(3) 6π (4) 12π

Answer (2)

Sol. $$\int_0^{\frac{\pi}{6}} \frac{\pi+4x^{11}}{1-\sin\left(|x|+\frac{\pi}{6}\right)}+\frac{\pi-4x^{11}}{1-\sin\left(|x|+\frac{\pi}{6}\right)}dx$$

$$=2\pi\int_0^{\frac{\pi}{6}} \frac{1}{1-\sin\left(x+\frac{\pi}{6}\right)}dx$$

$$=2\pi\int_0^{\frac{\pi}{6}} \frac{1+\sin\left(x+\frac{\pi}{6}\right)}{\cos^2\left(x+\frac{\pi}{6}\right)}dx$$

$$=2\pi\int_0^{\frac{\pi}{6}} \sec^2\left(x+\frac{\pi}{6}\right)+\tan\left(x+\frac{\pi}{6}\right)\sec\left(x+\frac{\pi}{6}\right)dx$$

$$\Rightarrow 2\pi\left[\tan\left(x+\frac{\pi}{6}\right)+\sec\left(x+\frac{\pi}{6}\right)\right]_0^{\frac{\pi}{6}}$$

$$\Rightarrow 4\pi$$

13. If $f(3) = 18$, $f'(3) = 0$ and $f''(3) = 4$. Then, the value of $\lim\limits_{x\to 1} \ln\left(\frac{f(x+2)}{f(3)}\right)^{\frac{18}{(x-1)^2}}$ is equal to

(1) 2 (2) 4

(3) 6 (4) 8

Answer (1)

Sol. $$\ln\left(\lim_{x\to 1}\left(\frac{f(x+2)}{f(3)}\right)^{\frac{18}{(x-1)^2}}\right)(1^\infty)$$

$$\ln\left(e^{\lim\limits_{x\to 1}\frac{18}{(x-1)^2}\left(\frac{f(x+2)}{f(3)}-1\right)}\right)$$

$$\ln\left(e^{\lim\limits_{x\to 1}\frac{f'(x+2)}{2(x-1)}}\right)$$

$$\ln\left(e^{\lim\limits_{x\to 1}\frac{f''(x+2)}{2}}\right)$$

$$\lim_{x\to 1}\frac{f''(3)}{2}=\frac{4}{2}=2$$

14. If the domain of the function $\cos^{-1}\left(\frac{2x-5}{11x-7}\right)+\sin^{-1}(2x^2-3x+1)$ is $[0,a]\cup\left[\frac{12}{13},b\right]$ then $\frac{1}{ab}$ is equal to

(1) –3

(2) 3

(3) 2

(4) 4

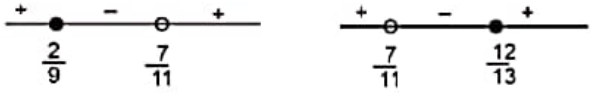
Answer (2)

Sol. $\cos^{-1}\left(\frac{2x-5}{11x-7}\right)$ is defined as

$$\frac{2x-5}{11x-7}\le 1, \frac{2x-5}{11x-7}\ge -1$$

$$\frac{-9x+2}{11x-7}\le 0, \frac{13x-12}{11x-7}\ge 0$$

$$\Rightarrow \frac{\left(x-\frac{2}{9}\right)}{\left(x-\frac{7}{11}\right)}\ge 0, \frac{\left(x-\frac{12}{13}\right)}{\left(x-\frac{7}{11}\right)}\ge 0$$



$$\Rightarrow\left[\left(-\infty,\frac{2}{9}\right]\cup\left(\frac{7}{11},\infty\right)\right]\cap\left[\left(-\infty,\frac{2}{11}\right)\cup\left[\frac{12}{13},\infty\right)\right]$$

$$\Rightarrow x\in\left(-\infty,\frac{2}{9}\right]\cup\left[\frac{12}{13},\infty\right)$$

Similarly, $\sin^{-1}(2x^2-3x+1)$ is defined

When $-1\le 2x^2-3x+1\le 1$

$$\Rightarrow 2x^2-3x\le 0 \Rightarrow x\left(x-\frac{3}{2}\right)\le 0$$

$$\Rightarrow x\in\left[0,\frac{3}{2}\right]$$

and $2x^2-3x+2\ge 0\Rightarrow x\in R \Rightarrow x\in\left[0,\frac{3}{2}\right]$

$\Rightarrow \cos^{-1}\left(\frac{2x-5}{11x-7}\right)+\sin^{-1}(2x^2-3x+1)$ is defined

for $\left[0,\frac{2}{9}\right]\cup\left[\frac{12}{13},\frac{3}{2}\right]$

$\Rightarrow ab=\frac{2}{9}\times\frac{3}{2}=\frac{1}{3}$

$\Rightarrow \frac{1}{ab}=3$

15. In binomial expansion of $(ax^2 + bx + c)(1 - 2x)^{26}$, the coefficients of x, x^2 and x^3 are -56, 0 and 0 respectively, then $(a + b + c)$ is equal to

(1) 1500 (2) 1403

(3) 1300 (4) 1483

Answer (2)

Sol. $(ax^2 + bx + c)(1 - 2x)^{26}$

$ax^2(1 - 2x)^{26} + bx(1 - 2x)^{26} + c(1 - 2x)^{26}$

Coefficient of

$x \Rightarrow (0 + b\cdot{}^{26}C_0 + c\cdot{}^{26}C_1(-2)x$

$= b - 52c$

Coefficient of

$x^2 = (a({}^{26}C_0) + b\cdot{}^{26}C_1(-2) + c\,{}^{26}C_2(-2)^2)x^2$

$= a - 52b + {}^{26}C_2 \times 4 = a - 52b + 1300c$

Coefficient of

$x^3 = [a\cdot{}^{26}C_1(-2) + b\cdot{}^{26}C_2(26)^2 + c\,{}^{26}C_3(-2)^3]$

$= -52a + 1300b - 20800c$

$\Rightarrow \quad b - 52c = -56$

$\Rightarrow \quad b - 52c = -56$

$a - 52b + 1300c = 0 \qquad \Rightarrow \quad a = 1300$

$-a + 25b - 400c = 0 \qquad b = 100$

$c = 3$

$\Rightarrow \quad a + b + c = 1403$

16. If $a_1 = 1$ and for $\forall n \geq 1$ $a_{n+1} = \frac{1}{2}a_n + \frac{2 - 2n - 1}{n^2(n+1)^2}$ then

$\left|\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right|$ is equal to

(1) 3 (2) 4

(3) 5 (4) 2

Answer (4)

Sol. $a_{n+1} = \frac{1}{2}a_n + \frac{1}{(n+1)^2} - \frac{\left((n+1)^2 - n^2\right)}{n^2(n+1)^2}$

$a_{n+1} = \frac{a_n}{2} + \frac{2}{(n+1)^2} - \frac{1}{n^2}$

$a_{n+1} - \frac{2}{(n+1)^2} = \frac{1}{2}\left(a_n - \frac{2}{n^2}\right)$

Let $b_n = a_n \frac{-2}{n^2}$

then $<b>$ is geometric progression with ratio $= \frac{1}{2}$

$b_1 = a_1 - \frac{2}{1} = 1 - 2 = (-1)$

$\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right) = \sum_{n=1}^{\infty} {}_n = \frac{(-1)}{1-\left(\frac{1}{2}\right)} = \frac{1}{1/2} = -2$

$\Rightarrow \left|\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right| = 2$

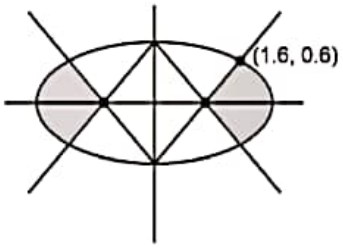
17. Area enclosed by $x^2 + 4y^2 \leq 4$, $y \leq |x| - 1$, $y \geq 1 - |x|$ is

(1) $4\sin^{-1}\left(\frac{3}{5}\right) + \frac{6}{5}$ (2) $\sin^{-1}\left(\frac{3}{5}\right) - \frac{6}{5}$

(3) $4\sin^{-1}\left(\frac{3}{5}\right) + \frac{12}{5}$ (4) $4\sin^{-1}\left(\frac{3}{5}\right) - \frac{6}{5}$

Answer (4)

Sol.



$$4\int_0^{0.6}\left[\sqrt{4-4y^2}-(1+y)\right]dy$$

$$I_1 = 4\int_0^{\frac{3}{5}}\sqrt{4-4y^2}dy, \text{ put } y=\sin\theta,\ dy=\cos\theta d\theta$$

$$\Rightarrow I_1 = \int_0^{\theta}2\cos^2\theta d\theta = \int_0^{\theta}(1+\cos^2\theta)d\theta = \left[\theta+\frac{\sin2\theta}{2}\right]_0^{\theta}$$

$$\therefore \sin = \frac{3}{5}, \cos\theta = \frac{4}{5}, \sin2\theta = \frac{24}{25}$$

$$\Rightarrow I_1 = \left(\sin^{-1}\frac{3}{5}+\frac{12}{25}\right), I_2 = \int_0^{\frac{3}{5}}(1+y)dy = \frac{39}{50}$$

$$\Rightarrow \text{Area} = 4\sin^{-1}\left(\frac{3}{5}\right)-\frac{6}{5}$$

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $x^2 + x + 1 = 0$, then

$$\left(x+\frac{1}{x}\right)^4+\left(x^2+\frac{1}{x^2}\right)^4+\left(x^3+\frac{1}{x^3}\right)^4+\ldots+\left(x^{25}+\frac{1}{x^{25}}\right)^4 \text{ is}$$

Answer (145)

Sol. $x^2 + x + 1 = 0 \begin{matrix}\nearrow w \\ \searrow w^2\end{matrix}$

$$\left(w+\frac{1}{w}\right)^4+\left(w^2+\frac{1}{w^2}\right)^4+\left(w^3+\frac{1}{w^3}\right)^4+\ldots.\left(w^{25}+\frac{1}{w^{25}}\right)^4$$

$$\sum\left(w^k+\frac{1}{w^k}\right)^4$$

$$K = 3x \Rightarrow w^{3x}+\frac{1}{w^{3x}} = 2$$

$$k \neq 3x \Rightarrow w^k+\frac{1}{w^k} = -1$$

$$\sum_{k=1}^{25}\left(w^k+\frac{1}{w^k}\right)^4 \Rightarrow 8\left(1+1+2^4\right)+1$$

$= 145$

22. The sum of roots of the equation $|x-1|^2-5|x-1|+6 = 0$ is

Answer (4)

Sol. $|x-1|2-5|x-1|+6=0$

Let $|x-1| = t$

$t^2 - 5 + 6 = 0$

$(t-3)(t-2) = 0$

$t = 2, 3$

$|x-1| = 2$ or $|x-1| = 3$

$x - 1 = \pm2$ or $x - 1 = \pm3$

$\Rightarrow x = 3, -1, 4, -2$

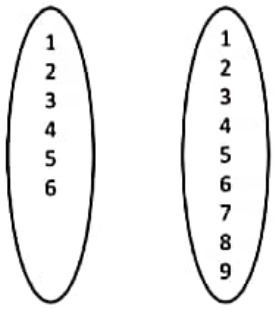
$\Rightarrow$ Sum $= 3 - 1 + 4 - 2$

$= 4$

23. If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{1, 2, 3, \ldots 8, 9\}$. Then the number of strictly increasing functions from $A \to B$ such that $f(i) \neq i\ \forall\ i = 1, 2, 3, 4, 5, 6$ are

Answer (28)

Sol.



Case (i) If $f(1) = 2$ then ${}^{7}C_5$ functions

Case (ii) $f(1) = 3$ then ${}^{6}C_5$

Case (iii) $f(1) = 4$ then ${}^{5}C_5$

$\Rightarrow 21 + 6 + 1 = 28$

24.

25.

❑ ❑ ❑